Debt dynamics and credit risk*

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Abstract

The dynamics of debt are crucial in structural models of credit risk and this paper provides an empirical examination of these dynamics. For US industrial firms, we find that the future level of debt of is negatively related to current leverage. Furthermore, when a firm experiences a negative shock to its equity, debt increases in the short run but declines in the long run, relative to a positive-shock firm. We incorporate these features into a structural model of credit risk and compare the model's ability to match the cross-section of US credit spreads with that of existing models. The model provides more accurate predictions of credit spreads, particularly for short-maturity investment grade debt.

Keywords: Structural Models, Debt Levels, Default Rates, Default Boundary, Credit Risk;

JEL: C23; G12

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1. Introduction

Structural models of credit risk are widely used in financial economics.[1] The models assume some dynamics for firm value and that the firm defaults the first time firm value crosses a default boundary. The default boundary is a key component and is typically a function, exogeneously given or endogeneously derived, of the current level of debt. While there is an extensive literature on the dynamics of *leverage*, there is little empirical evidence on the dynamics of the *level* of debt and their implications for debt pricing. In this paper we provide such evidence.

We examine the dynamics of the notional amount of debt for 17,650 US industrial firms in the period 1965–2017. The average annual growth rate in log-debt is 10%, and when we sort firms according to leverage (debt over firm value), there is a monotone negative relation between leverage and the growth rate of debt; the average growth rate is 23% (-17%) for firms with a leverage less than 20% (more than 80%).

We then group firms with similar leverage and within each group sort firms into high or low equity return depending on whether their *future* equity returns are above or below the median. We find that over the period for which we have conditioned on equity returns, firms with *low* equity returns take on *more* debt than firms with high equity returns, and this is repeated systematically across different levels of initial leverage. In the years following the period over which equity returns are conditioned, high return firms increase debt more than low return firms and end up with more debt. To illustrate the effect, Figure 1 shows the future level of debt for firms with high and low equity returns, where firms are sorted according to their 1-, 3-, 5-, and 7-year equity return (but not sorted on initial leverage).

We next incorporate these stylized facts about the evolution of the level of debt into models of credit risk. Our focus is on the dynamics of the level of debt, and we therefore investigate models with different assumptions about debt dynamics while keeping the specification of firm value dynamics and risk premiums standard. Specifically, we use benchmark assumptions

[1] Examples include research on the equity premium (Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2009)), firms' financing policies (Chen (2010)), bond premiums (Gilchrist and Zakrajsek (2012)), rollover risk (He and Xiong (2012)), liquidity risk (He and Milbradt (2014)), firms' investment decisions (Kuehn and Schmid (2014)), and default prediction (Gomes, Grotteria, and Wachter (2019)). See Sundaresan (2013) for more examples.

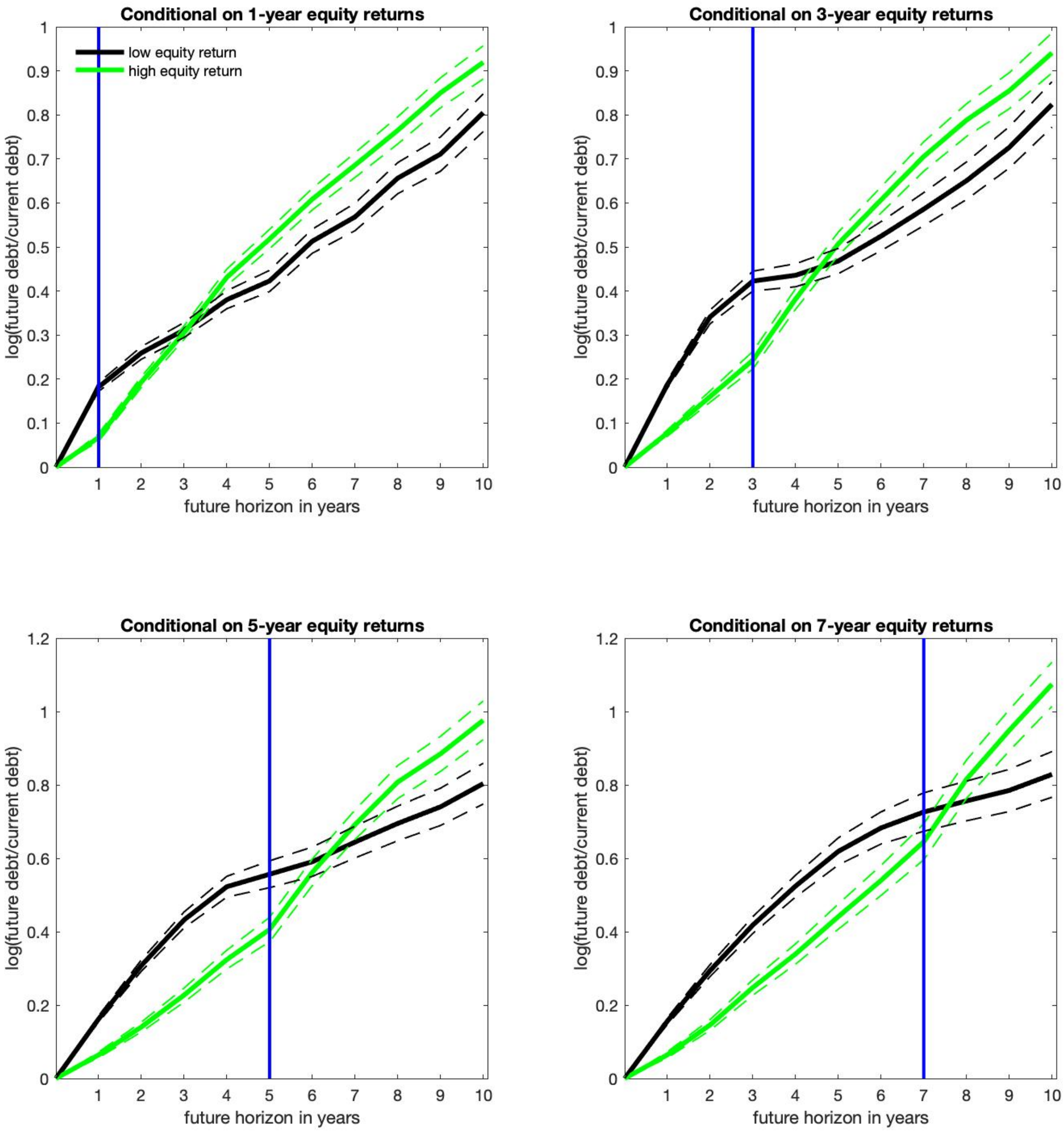

Fig. 1 *Log ratio of future debt relative to current debt conditional on future equity returns.* For firm i, year t, and horizons 1,...,10, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample, we calculate the future N-year equity return between t and $t+N$ and label firms with a return higher (lower) than the median between t and $t+N$ 'High (Low) future equity return' firms. The figures show the average log-ratio for high and low future equity return firms for N=1, 3, 5, and 7 years. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1965–2017.

in the literature and assume that firm value follows a Geometric Brownian Motion and that the asset Sharpe ratio is constant.

The models of debt dynamics we investigate are two well-known models as well as a new model we propose. The first model is the Black and Cox (1976) model where it is assumed that the growth rate of log-debt is constant. Most often, the model is used with the assumption that the growth rate is zero[2] and we therefore explore the model with both a zero growth rate and a growth rate estimated from historical data. The second model is the stationary leverage model of Collin-Dufresne and Goldstein (2001) where leverage mean-reverts towards a long-run target leverage. In this model, the local change in the level of debt is a deterministic function of leverage. Finally, we propose a fourth model, the stochastic debt model, where leverage mean-reverts towards a long-run target leverage – as in Collin-Dufresne and Goldstein (2001) 's model – but where, motivated by the dynamics of debt illustrated in Figure 1, the local change in the level of debt is exposed to a random shock that is correlated with the shock to firm value.

We estimate the parameters describing debt dynamics in these models by minimising the difference between historical and model-implied changes in the level of debt, and investigate how well they capture the documented debt dynamics. We find that the stochastic debt model is the only model that captures the main documented features of debt dynamics. Because leverage in the model is mean reverting, it captures the average growth rate in debt and the negative long-run relation between debt growth and leverage. Further, because the estimated correlation between shocks to firm value and the debt level is estimated to be negative, it implies a negative short-run relation between shocks to equity and the debt level that is consistent with the evidence in Figure 1. In contrast the Black-Cox model implies no short-run relation while the stationary leverage model counterfactually implies a positive relation.

Using US corporate bond yield data for the period 1988–2018, we calibrate the models to match historical default rates and investigate the average pricing errors of the models both for individual transactions level and for average monthly spreads. The stochastic debt model

[2]For recent examples see Feldhütter and Schaefer (2018), Bai, Goldstein, and Yang (2019), and Huang, Nozawa, and Shi (2020)

has the smallest average pricing errors and this is due to more accurate pricing of short-term bonds. The reason for the more accurate pricing is that the stochastic debt model implies higher spreads for safe firms and lower spreads for risky firms compared to the other models, particularly for short-term bonds. This is the case in the cross-section as well as in the time series where the stochastic debt model predicts higher monthly spreads in calm periods and lower monthly spreads in volatile periods.

The firm defaults when leverage hits a boundary (from above) and the distinct predictions of the stochastic debt model is due to a higher volatility of leverage: in contrast to the standard models where leverage volatility is equal to firm value volatility, the stochastic debt model has both firm value volatility and debt level volatility contributing to leverage volatility. Higher leverage volatility in turn implies that the sensitivity of spreads to changes in leverage becomes smaller. The model is calibrated to match historical default rates by estimating the default boundary and at an average leverage default probabilities and thus spread predictions are similar in all the models (since leverage volatility in the stochastic debt model is higher the calibrated default boundary is lower). However, since the sensitivity of spreads to leverage changes is lower in the stochastic debt model, firms with low (high) leverage have higher (lower) spreads in the stochastic debt model compared to the other models.

Finally, we provide further evidence on the potential existence of a "credit spread puzzle", the finding that average investment grade spreads are too low compared to actual spreads. Feldhütter and Schaefer (2018) (FS) show that previous literature finding a puzzle relies on statistically imprecise historical default rates. They propose a statistically more precise approach that uses a cross section of default rates and find no evidence of the puzzle. FS equate the book value of debt as a proxy for the market value and Bai, Goldstein, and Yang (2019) (BGY) point out that this theoretically incorrect. BGY suggest using corporate bond prices to calculate the market value of debt and using the approach in FS they find that the puzzle re-appears when doing so. We shed further light on the important point made by BGY by a) using more accurate debt market value calculations from Bretscher, Feldhütter, Kane, and Schmid (2020) that take into account both bond and bank debt prices, and b) excluding C-rated firms (which BGY show has significant deviations between market and book values) in the calibration to historical default rates. In both cases and across a range of models, we find robust evidence that there is no credit spread puzzle.

We are not aware of any papers that both provide an empirical analysis of the dynamics of the level of debt and then also use these dynamics to distinguish between different structural models of credit risk. Some papers test a number of different structural models: for example, Huang and Huang (2012) investigate different structural models calibrated to match the historical default rate exactly at each maturity and rating. We calibrate to a cross-section of default rates, with each default rate measured with error, and this allows us to investigate the model-implied term structures of default rates and bond spreads. Eom, Helwege, and Huang (2004) [EHH] test structural models with different dynamics for the default boundary and report that most of the models overpredict spreads. They do not calibrate to historical default rates, and assume that the default point is equal to the face value of debt. We estimate the default point by calibrating to historical default rates and find it to be substantially below the face value, suggesting that the overpricing EHH find is due to misestimating the default point, not a failure of the structural models per se. Huang, Shi, and Zhou (2019) use GMM to estimate a range of structural models and use equity volatility and CDS spreads as GMM moments. They focus on the first and second moments of equity returns and CDS spreads in a shorter period 2002-2004, while we focus on debt dynamics and corporate bond spreads in a longer period 1988-2018. Bai, Goldstein, and Yang (2019) reject the joint assumption of the firm value following a Geometric Brownian Motion and the level of debt being constant. They suggest firm value dynamics that differ from standard models while, here, we suggest different debt value dynamics. Dorfleitner, Schneider, and Veza (2011) propose a general specification of the default boundary and calibrate the model to CDS prices of two firms, but do not conduct a large empirical examination as we do. Also, Hackbarth and Kitapbayev (2019) derive default probabilities in a model with an increasing default boundary but do not provide an empirical investigation as we do. Finally, Flannery, Nikolova, and Oztekin (2012) investigate empirically the relation between future changes in leverage and current credit spreads, but do not isolate the contribution of the level of debt or incorporate their findings in a structural model.

2 Data

In our analysis we use firm variables (such as leverage and equity volatility), yield spreads on corporate bonds along with individual bond information, and calculate historical default rates. We focus on the US market and use as main data sources: firm variables from CRSP/Compustat, corporate bond quotes from the Lehman Brothers and Merrill Lynch databases, corporate bond transaction prices from the Trade Reporting and Compliance Engine (TRACE), bond information from the Mergent Fixed Income Securities and default data from Moody's Default and Recovery Database. The data sources are well known and used in a large number of studies and below we provide a brief description of each.

Firm variables

Firm variables are collected in the CRSP/Compustat Merged Database and computed as in Feldhütter and Schaefer (2018). For a given firm and year the *nominal amount of debt* is the debt in current liabilities (DLCQ) plus long-term debt (DLTTQ) in the fourth quarter of the year. We analyse industrial firms and therefore exclude utilities (SIC codes 4900-4949) and financials (SIC codes 6000-6999). The data period is 1965-2017. We calculate the *market value of equity* as the number of shares outstanding (CSHOQ) times the closing share price in the quarter (PRCC). The *leverage ratio* is calculated as (nominal amount of debt)/(market value of equity + nominal amount of debt). The number of firm-year observations with both the level of debt and market value of equity available is 185,359 and the number of firms is 17,650.

Equity volatility $\sigma_{E,t}$ is computed as $\sqrt{255}$ times the standard deviation of daily stock returns in the past three years. If there are no return observations on more than half the days in the three-year window, we do not calculate equity volatility. We follow Feldhütter and Schaefer (2018) and calculate *asset volatility* at time t as $(1 - L_t)\sigma_{E,t}$ and multiply this by 1 if $L_t < 0.25$, 1.05 if $0.25 < L_t \leq 0.35$, 1.10 if $0.35 < L_t \leq 0.45$, 1.20 if $0.45 < L_t \leq 0.55$, 1.40 if $0.55 < L_t \leq 0.75$, and 1.80 if $L_t > 0.75$.

The *payout rate* is the total outflow to stake holders divided by firm value. This is computed as the sum of the previous year's interest payments, dividend payments, and net stock repurchases divided by the sum of market value of equity and book value of debt. The payout ratio is winsorized at 0.13 as in Feldhütter and Schaefer (2018).

Corporate bond yield spreads

We use several sources to arrive at our U.S. corporate bond data set for the period April 1988 to March 2018. For the period April 1988 to December 1996 we use monthly data from the Lehman Brothers Fixed Income Database and include only actual quotes. For the period January 1997 to June 2002, we use quotes provided by Merrill Lynch (ML) on all corporate bonds included in the ML investment grade and high-yield indices. For each bond-month we use the last quote in the month. Feldhütter and Schaefer (2018) show that there is a significant bid-bias in bond quotes for short-maturity bonds and we therefore follow Feldhütter and Schaefer (2018) and exclude ML and Lehman quotes for bonds with a maturity less than three years. For the period July 2002-June 2017 we use transactions data from Enhanced TRACE and for the period July 2017-March 2018 transactions data from standard TRACE. We filter transactions according to Dick-Nielsen (2009, 2014) and focus on transactions with a volume of $100,000 or more. When using TRACE, we calculate one yield observation for each bond-month by computing the median yield for the bond in the month.[3] When we match yield observations to firm variables, we use firm variables from the day the median is observed.

Bond information

We obtain bond information from the Mergent Fixed Income Securities Database (FISD) and limit the sample to senior unsecured fixed rate or zero coupon bonds. We exclude bonds that are callable, convertible, putable, perpetual, foreign denominated, Yankee, or have sinking fund provisions.[4] We use only bonds issued by industrial firms and restrict our sample to bonds with a maturity of less than 20 years to be consistent with the maturities of the default rates we use as part of the estimation. After we merge the bond data with firm variables, the number of observations is 119,765. We winsorize spreads at the 1% and 99% level.

Riskfree rates

[3] If there are N observations in a month where N is even, we sort the observations increasingly and use the $N/2$'th observation.

[4] For bond rating, we use the lower of Moody's rating and S&P's rating. If only one of the two rating agencies have rated the bond, we use that rating. We track rating changes on a bond, so the same bond can appear in several rating categories over time.

As in Feldhütter and Schaefer (2018), Bai, Goldstein, and Yang (2019), and others we calculate corporate bond yield spreads relative to the swap rate and use on a given date the available rates among the 1-week, 1-month, 2-month, and 3-month LIBOR and 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20-year swap rates and linearly interpolate to obtain a swap rate at the exact maturity of the bond. Before 1998 the longest swap maturity is 10 years so in the early period we use, for longer maturities, the Treasury CMT rate plus the swap spread at the longest maturity for which there is a swap rate.

Default data

Data on defaults are from Moody's Analytics' Default and Recovery Database (DRD v2.0). In the period from 1919 to 2018, the database contains rating history for 27,750 unique firms and 11,024 default events. There are four events that constitute a debt default: a missed interest or principal payment, a bankruptcy filing, a distressed exchange, and a change in the payment terms of a credit agreement or indenture that results in a diminished financial obligation. Soft defaults ('dividend omission' and 'BFSR default') appear in the database, but we follow Moody's and exclude these when calculating default rates. The database includes information on the (latest) company industry and domicile.

We set the recovery rate to 33.48% which is Moody's (2018a)'s average recovery rate, as measured by post-default trading prices, for senior unsecured bonds for the period 1983-2017.

3 New facts about the dynamics of debt

In structural models of credit risk, the amount of debt plays a key role as it determines the firm's default boundary and thus its default probability and credit spread. Moreover, in pricing bonds or estimating default probabilities, it is the amount of debt *at the time of default* which is critical rather than the amount of debt at the price observation date. Thus, in order to price bonds or estimate default probabilities, we need to characterise the dynamics of a firm's debt level. The empirical evidence on this point is limited and in this section we present new facts.

We first investigate how the level of debt changes over time for the average firm. For firm i in year t we define the nominal gross amount of debt as $D_{i,t}$. For a future horizon T

(measured in years) we calculate the total growth rate of firm i's debt as:

$$R_{it}^{T} = \log\left(\frac{D_{i,t+T}}{D_{i,t}}\right) \tag{1}$$

and discard the observation if the amount of debt for firm i at time $t+T$ is not reported.[5] We measure the log growth rate of debt rather than the simple ratio for two reasons. First, log is more robust to outliers and, second, in our proposed model, the level of debt is defined in logs. Our estimate of the average growth rate for horizon T is

$$R^{T} = \frac{1}{N^{T}} \sum_{i} \sum_{t} R_{it}^{T} \tag{2}$$

where N^{T} is the total number of observations of R_{it}^{T}.

Table 1 shows the average log debt ratio in our sample period 1965-2017. On average, the amount of debt increases substantially. For all firms, the face value of debt after 10 years is 134% higher and the increase is highly statistically significant. This fact may seem obvious in the sense that, since asset values grow over time, if nominal debt did not also grow the average level of corporate leverage would tend to zero. Yet, it is often assumed in the literature on credit risk that the growth rate is zero.

To examine how future debt depends on how much debt the firm already has, Figure 2 and Table 1 show how the future amount of debt depends on firms' initial leverage. Firms with less leverage have a higher growth rate of future debt. For example, for firms with a leverage between 0 and 20% the nominal amount of debt over a 10-year horizon increases on average by 331% while for firms with a leverage between 60 and 80% there is only a slight increase of 7%.[6] We also see that highly levered firms reduce their debt over the short horizon. For example, firms with a leverage of more than 80% decrease their nominal debt by 20% on average over a horizon of ten years.[7] Therefore,

[5] We restrict our analysis to firms with a positive amount of debt which is the case for 86.8% of the firm-year observations.

[6] Table 1 shows standard errors on the growth rates and the differences are highly statistically significant: assuming independence, the variance of the difference in means between two groups can be calculated as the sum of the variances of each group's mean.

[7] One may worry about a simple average across time and firms for the following reason. If, in recessions, firms are more highly leveraged and have lower debt growth rates, the low debt growth rate of highly

Key fact 1: There is a negative relation between current leverage and the future growth rate of nominal debt.

We next sort firms according to future equity returns. Specifically, at time t and for a given leverage group, we calculate the firms' three-year future equity returns, $\frac{P_{t+3}-P_t}{P_t}$ where P_t is the share price at the end of year t, and sort firms into two groups according to whether their return is below or above the median.[8] For example, if there were 200 firms in 1995 with a leverage between 0.4 and 0.6, we calculate their equity returns between 1995 and 1998, and the 100 firms with equity returns above (below) the median return are classified as high (low) equity return. We repeat this for the other years in the sample to arrive at our final sample of high and low equity return firm-years (firms may of course switch between high and low return over the years). Figure 3 shows the average change in debt over six years: three years either side of the three-year horizon over which equity returns are measured. The figure shows that, on average, firms that experience low equity returns over the next three years take on *more* debt than firms with the same initial leverage that experience high equity returns, and this is the case for all initial leverage ratios.[9] Table 2 shows that the differences are statistically highly significant. If firms always adjusted their debt levels so as to maintain a target leverage ratio, we would see the opposite, namely that firms with low equity returns would have lower rates of debt increase. This leads to

Key Fact 2: Short-run changes in firms' debt levels are negatively correlated with their short-run equity returns.

Table 2 shows how debt levels change not only during the three years of the equity shock, but also in the years after the shock. The table gives the average change in log debt and shows that although a negative equity shock leads to a higher short-run growth rate in debt, in the long run it leads to a lower growth rate. For example, firms with an initial leverage between 0.4 and 0.6 that experience a positive (negative) three-year equity shock change

leveraged firms may partially be due to the low debt growth rates in recessions. To address this concern we have also, for each leverage group, calculated the average debt growth rate for each year in the sample and then calculated the average across years. Results are very similar and available on request.

[8]One may worry about stock splits when using changes in the stock price. However, results are similar if we restrict the sample to firms where the number of shares outstanding between time t and $t+3$ does not change by more than $\pm 10\%$ and therefore stock splits are not driving our results.

[9]The conclusion is very similar if we condition on equity returns over a different horizon as Figure 1 shows.

their debt level during those three years by -5% (+8%) on average and by 52% (25%) after ten years, i.e. seven years after the equity shock. We therefore have

Key Fact 3: Firms' long-run debt levels are positively correlated with firms' short-run equity returns.

We are mainly interested in total debt as this is the variable most commonly used when defining the default boundary in structural models. However, some researchers use KMV's Expected Default Frequencies and KMV uses the current value of short-term debt plus a half of long-term debt as the default point. To see if our finding that low equity return firms take on more debt depends on our definition of total debt, Figure 4 shows the future growth of short- and long-term debt separately. We see that both types of debt show the same pattern, namely that low equity returrn firms increase debt in the short run and decrease debt in the long-run. Furthermore, the figure shows that low equity return firms accumulate less cash, so they are not hoarding cash when hit by a bad shock.[10] The figure also shows that our main result is very similar if we define leverage as total debt minus cash. Finally, the figure shows the future change in leverage. We see that the joint effect of a negative equity shock and an increase in debt leads to a substantial increase in leverage that only slowly reverts back after the shock.

With these stylized facts as yardsticks, we now compare the ability of structural models of credit risk to match actual debt dynamics. We immediately note that many models in the literature assume, counterfactually, that the amount of debt is constant[11] and this is an area in which structural models could potentially be significantly improved.

4 Structural models and their dynamics of debt

In this section we first discuss the four structural models we implement, three that are well-known from the literature and a new model that we propose. We focus on the specification of the default boundary; the specification of asset value dynamics and risk premium are

[10] We use 'Cash and short-term investments ' (CHE) as cash. Results are very similar if we use 'Cash' (CH) instead.

[11] See for example Merton (1974), Leland (1994), Leland and Toft (1996), Cremers, Driessen, and Maenhout (2008), He and Milbradt (2014), Feldhütter and Schaefer (2018), Bai, Goldstein, and Yang (2019), and **?**).

standard. Then, we describe how we estimate the parameters of the models and compare their debt level dynamics with those documented in the previous section.

4.1 Structural models

We assume that the firm's asset value follows a Geometric Brownian Motion under the natural measure,

$$\frac{dV_t}{V_t} = (\mu - \delta)dt + \sigma dW_t^P \tag{3}$$

where δ is the payout rate to debt and equity holders, μ is the expected return on the firm's assets and σ is the volatility of returns on the asset.

The firm defaults the first time the value of the firm hits the default boundary (from above). The default boundary is a constant fraction, d, of the face value of debt, K_t, and so the default time of the firm is

$$\tau = \inf\{t|V_t \leq d \times K_t\}. \tag{4}$$

The models we examine differ only in their assumptions about the dynamics of the firm's debt, K_t. We denote log-firm value as $v_t = \log(V_t)$ and log-debt as $k_t = \log(K_t)$, and so the default time may also be written as:

$$\tau = \inf\{t|l_t \geq -log(d)\} \tag{5}$$

where log-leverage is $l_t = k_t - v_t$.

Following Eom, Helwege, and Huang (2004), Bao (2009), Huang and Huang (2012), Feldhütter and Schaefer (2018), **?**), and others we assume that if default occurs, investors receive at maturity a fraction of the originally promised face value, but now with certainty. Assuming the bond is a zero-coupon bond, the credit spread, s, is then calculated as:

$$s = y - r = -\frac{1}{T}\log[1 - (1 - R)\pi^Q(T)] \tag{6}$$

where y is the yield-to-maturity, r is the riskless rate, R is the recovery rate, T is the bond maturity and $\pi^Q(T)$ is the risk-neutral default probability. We next describe the assumptions about the evolution of debt in the models we implement. Appendix A provides formulae for default probabilities and the expected level of debt for each model.

Constant level of debt [BC-0G]

The most common assumption for the dynamics of debt is that the level of debt remains constant (see footnote 11 for examples), i.e.

$$k_t = k_0 \text{ for all } t > 0. \tag{7}$$

This corresponds to the Black and Cox (1976) model with no growth in the default boundary, and we refer to this model as BC-0G.

Constant growth in debt [BC]

In the Black and Cox (1976) model, the level of debt is $K_t = K_0 e^{\gamma t}$, i.e.

$$k_t = k_0 + \gamma t \tag{8}$$

where $\gamma > 0$. In this case, the level of debt increases deterministically over time. Given that it is well-known that, on average, firms increase their debt over time, it is perhaps surprising that this model is never used in the literature.[12]

Deterministic debt adjustment [CDG]

Collin-Dufresne and Goldstein (2001) propose a structural model in which leverage is stationary and the adjustment to target leverage is locally deterministic. Specifically, the dynamics of the log-debt level, k_t, in the CDG model are given by

$$dk_t = \lambda(\nu - l_t)dt \tag{9}$$

[12]Bao (2009) and Feldhütter and Schaefer (2018) implement a model they refer to as the Black-Cox model, but in our terminology this is the constant level of debt model.

where $\lambda > 0$ and l_t is log-leverage. If log-leverage is above (below) a target ν, the firm reduces (increases) the level of debt. In the long run, the expected level of debt is proportional to firm value. This model has been used in Eom, Helwege, and Huang (2004) and Huang and Huang (2012), among others.

Stochastic debt adjustment [SD]

To accommodate the features of debt dynamics that are documented in Section 3, we propose a model of the dynamics of the debt level given by:

$$dk_t = \lambda(\nu - l_t)dt + \sigma_k dW_{k,t} \tag{10}$$

where the correlation between the shock to debt, W_k, and the shock to firm value, W^P, is ρ. Mean reversion, $\lambda(\nu - l_t)dt$, implies that the expected level of debt is proportional to firm value in the long run, as is the case in the CDG model. The presence of the stochastic component, $\sigma_k dW_{k,t}$, means that, in the short run, leverage deviates from a deterministic drift towards the firm's target leverage ratio.

4.2 Estimation of debt dynamics parameters

We estimate the parameters of a firm's debt dynamics by matching model-implied debt dynamics to the historical estimates documented in Section 3. Specifically, we denote by $\overline{D}^g_{i,T}$ the historical average T-year log-growth in debt for firms with an initial leverage in leverage range L_i. We use initial leverage ranges $L_1 = [0; 0.2], L_2 = [0.2; 0.4], L_3 = [0.4; 0.6], L_4 = [0.6; 0.8]$, and $L_5 = [0.8; 1]$, and the historical log-growth rates are given in Table 1. Furthermore, denote the historical average T-year log-growth in debt for firms with an initial leverage that is in leverage group L_i, and experiences a three-year equity shock above (below) the median by $\overline{D}^{g,H}_{i,T}$ ($\overline{D}^{g,L}_{i,T}$) and the difference by $\overline{\Delta D}^g_{i,T} = \overline{D}^{g,H}_{i,T} - \overline{D}^{g,L}_{i,T}$. The historical conditional growth rates are given in Table 2.

All models have the same dynamics for the value of the firm (equation (3)) and we set the parameters to average values (estimated later and given in Table A5): $\hat{\Theta}^P_v = (\mu, \delta, \sigma) =$

$(0.0996, 0.044, 0.24)$.[13] The BC-0G model requires no debt-dynamics parameters. For the other models, the debt dynamics parameters are given as $\Theta^{\mathrm{BC}} = \gamma, \Theta^{\mathrm{CDG}} = (\lambda, \nu)$, and $\Theta_k^{\mathrm{SD}} = (\lambda, \nu, \sigma_k, \rho)$. We estimate these parameters for each model by minimizing the weighted squared differences between historical debt growth rates and model-implied debt growth rates where the weights are the precision with which the historical growth rates are estimated:

$$\min_{\Theta_k} \sum_{i=1}^{5} \sum_{T=1}^{10} \Big(\frac{1}{SD_{i,T}^{\overline{D}}} (\overline{D}_{i,T}^{g} - D_T^g(L_i^m, \hat{\Theta}_v^P, \Theta_k))^2 + \frac{1}{SD_{i,T}^{\overline{\Delta D}}} (\overline{\Delta D}_{i,T}^{g} - \Delta D_T^g(L_i^m, \hat{\Theta}_v^P, \Theta_k))^2 \Big). \quad (11)$$

Here $SD_{i,T}^{\overline{D}}$ are the standard errors given in Table 1, $SD_{i,T}^{\overline{\Delta D}}$ are the standard errors given in Table 2, L_i^m is the mid point in leverage interval L_i, $D_T^g(L_i^m, \hat{\Theta}_v^P, \Theta_k)$ is the model-implied growth rate in debt, and $\Delta D_T^g(L_i^m, \hat{\Theta}_v^P, \Theta_k)$ is the model-implied difference in growth rates between low and high 3-year equity shock firms.[14]

4.3 Model-implied debt dynamics

For each of the four models, Figure 5 shows the expected value of future debt, relative to current debt for horizons one to ten years along with the corresponding values from the data. The BC-0G model assumes that the level of debt remains constant which is clearly counterfactual as Figure 5 shows. The growth rate in the BC model is estimated to be $\hat{\gamma} = 0.0534$ and the model is an improvement relative to the BC-0G model because it at least implies an increase in the level of debt over time. However, the model does not capture cross-sectional differences in the growth rate of debt as Figure 5 shows: there is a large difference in the average 20-year historical average growth rate of firms with low and high leverage firms while in both cases the BC model implies an identical increase. (Table A1 in the Appendix shows the values).

The debt parameters of the CDG model are estimated to be $(\hat{\lambda}, \hat{\nu}) = (0.1232, -0.5414)$. The half-life of the leverage adjustment is $\log(2)/0.1232 = 5.6$ years, and thus firms slowly

[13]Specifically, the average riskfree rate is $r = 0.04679$ and with a Sharpe ratio of $\theta = 0.22$ we have that $\mu = r + \theta\sigma = 0.04679 + 0.22 * 0.24 = 0.0996$.

[14]The model-implied growth rate in debt is 0 in the BC-0G model, γt in the BC model, and given in equation (29) for the CDG and SD models. The model-implied difference in growth rates between low and high equity shock firms is 0 in the BC-0G and BC models and given in equation (43) for the CDG and SD models.

adjust their leverage to changes in firm value. The estimate of ν corresponds to a target leverage ratio of 0.48.[15] Figure 5 shows that the CDG and SD models fit the cross-section of historical debt changes substantially better than the BC-0G and BC models.

Figure 6 show the historical and model-implied difference between the debt level of firms with an above-median and below-median value shock (Table A2 in the Appendix shows the values). We see that regardless of the initial leverage, firms with a positive value shock increase debt more in the CDG model than firms with a negative equity shock. This is inconsistent with actual firm behavior where a positive value shock leads to a smaller increase in the debt level compared to a negative value shock. In this regard, the predictions of the CDG model are worse than the BC and BC-0G models, where the prediction is no difference between high and low shock firms. Thus, the CDG model implies that firms “pull away” from default during a negative shock by reducing debt – relative to firms with a positive shock – while the actual behavior of firms is to increase debt when a negative shock occurs and so that firms may “spiral” towards default.

The SD model has four parameters $(\lambda, \nu, \sigma_k, \rho)$. We estimate the parameters by matching model-implied debt change behavior to historical debt change behavior and equations (29) and (43) in the Appendix show that σ_k and ρ are not separately identified; only the product $\sigma_k\rho$ is identified. The estimates of the three parameters are $(\hat{\lambda}, \hat{\nu}, \hat{\sigma_k\rho}) = (0.1451, -0.6057, -0.1322)$. The estimates of λ and ν are similar to those in the CDG model, and likewise, as Figure 5 shows, the average future level of debt, conditional on initial leverage, is also similar.

While the SD and CDG models imply similar future expected levels of debt, the models imply markedly different debt levels for firms experiencing a positive or negative value shock. Figure 6 shows that while the SD model implies more debt issuance by negative-shock firms relative to positive-shock firms, the CDG model implies the opposite. Thus, the SD model captures the “debt-spiralling” behavior of firms. The reason for this behavior in the model is that ρ is negative (since $\sigma_k\rho$ is negative) and therefore a negative shock to firm value implies a positive shock to debt issuance.

The speed of mean reversion of corporate leverage has been investigated extensively in the

[15]Equation (26) shows that the *log* target leverage is $\bar{l} = \nu - \frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda} = -0.5414 - \frac{0.0996-0.044-\frac{0.24^2}{2}}{0.1451} = -0.7261$.

literature and Frank and Goyal (2008) point out that the magnitude of the mean reversion is not a settled issue. Examples of estimates are 0.07-0.15 (Fama and French (2002)), 0.17-0.23 (Huang and Ritter (2009)), 0.13-0.39 (Lemmon, Roberts, and Zender (2008)), and 0.34 (Flannery and Rangan (2006)). Our estimate of 0.1451 is within, but in the lower range of, the range of estimates found in the literature.[16]

In the next part of the paper we examine calibrate the models to historical default rates and since the SD model σ_k and ρ have separate effects on default rates, we provide separate estimates as follows. We assume that firms with high asset volatility, σ, have high debt volatility, σ_k[17], and set σ_k to be proportional to σ for firm i. We estimate the proportionality factor by estimating the standard deviation of yearly changes in log-debt and dividing this by the standard deviation of yearly changes in log-firm value (where firm value is defined as the value of equity plus the face value of debt). In our sample this ratio is 0.8325. Since $\hat{\sigma_k \rho} = -0.1322$ and we have used $\sigma = 0.24$ in our estimation the implied correlation is $\hat{\rho} = -0.6617$. Thus, for every firm i in our sample we use $\rho = -0.6617$ and $\sigma_k^i = 0.8325\sigma^i$.

5 Implementation of models

We implement the four different structural models described in Section 4.1. For each model, we assume that the parameters for debt adjustment are common to all firms and set the values to those estimated in Section 4.3. For the deterministic and stochastic debt adjustment models, we also implement versions where we allow the target leverage ratio to be firm-specific. In this case, we calculate for a given firm the average historical log-leverage $\hat{l^i}$ and use this as the target log-leverage.[18] This is motivated by Lemmon, Roberts, and Zender

[16]When estimating the (common) mean reversion we assume a common target leverage. Flannery and Rangan (2006) and Lemmon, Roberts, and Zender (2008) find that including firm-specific heterogeneity in the estimation increases the estimate of speed of mean reversion and one may therefore hypothesise that different values of ν may lead to substantially higher estimates of λ. To examine this we reestimate λ and $\sigma_k\rho$ while holding the target leverage fixed at different values. We find that a target leverage fixed in the range 30-60% (0-100%) produces a mean reversion in the range 0.11-0.17 (0.00-0.17), which suggests that allowing for a firm-specific ν would not materially increase the speed of mean reversion estimate.

[17]If we define the yearly change in log-debt level for firm i in year t as $\Delta k_{i,t}$ and the yearly change in log-firm value, defined as equity value plus the face value of debt, as $\Delta v_{i,t}$, we have $corr(|\Delta k_{i,t}|, |\Delta v_{i,t}|) = 0.25$.

[18]In the models, the parameter of interest ν is different from target leverage. Equation (26) shows that the relation between ν and target leverage $\bar{l}$ is $\bar{l} = \nu - \frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda}$. We use a common adjustment $\frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda}$ for all firms

(2008), Huang and Ritter (2009), and others finding that firms have a target leverage and this target is firm-specific, stable and only to a lesser extent explained by firm characteristics or macroeconomic factors.

A large part of the literature that investigates the ability of structural models to price corporate debt matches models to a single historical default rate at a given maturity and for a specific rating. Feldhütter and Schaefer (2018) show that this results in a very noisy estimate of the default probability at this maturity and rating class and show further that matching models to default rates across maturities and ratings vastly improves the precision of default probability estimates. We therefore use their approach and extract a default boundary – d in equation (4) – common for all firms, that provides the best fit to the cross-section of historical default rates.

Specifically, for each model, we find d by the following procedure. For each observed spread in the data sample on bond i with a time-to-maturity T issued by firm j on date t, we calculate the firm's T-year default probability $\pi^P(dL_{jt}, \Theta^P_{jt}, T)$ where L_{jt} is the time-t estimate of the firm's leverage ratio and Θ^P_{jt} is a vector containing the relevant parameters for the specific model. Summary statistics of the bond sample are given in Table 3 and estimated firm values in Table A5. To calculate the drift of firm value μ_{jt} we assume a constant Sharpe ratio θ such that $\mu_{jt} = \theta\sigma_j + r_t^T - \delta_{jt}$, where r_t^T is the T-year riskfree rate and use Chen, Collin-Dufresne, and Goldstein (2009)'s estimate of the Sharpe ratio of 0.22.

For a given rating a and maturity T - rounded up to the nearest integer year - we find all bond observations in the sample with the corresponding rating and maturity. For a given calendar year y we calculate the average default probability $\overline{\pi}^P_{y,aT}(d)$ and we then calculate the overall average default probability for rating a and maturity T, $\overline{\pi}^P_{aT}(d)$, by computing the mean across the N years, $\overline{\pi}^P_{aT}(d) = \frac{1}{N}\sum_{y=1}^{N}\overline{\pi}^P_{y,aT}(d)$. We denote by $\hat{\pi}^P_{aT}$ the corresponding historical default frequency. For rating categories AAA, AA, A, BBB, BB, and B and horizons of 1-20 years we find the value of d that minimizes the sum of absolute

given as $\frac{0.0996-0.044-\frac{0.24^2}{2}}{0.1232} = 0.2175$ in the deterministic adjustment model and $\frac{0.0996-0.044-\frac{0.24^2}{2}}{0.1451} = 0.1847$ in the stochastic adjustment model.

differences between the annualized historical and model-implied default rates by solving

$$\min_{\{d\}} \sum_{a=AAA}^{B} \sum_{T=1}^{20} \frac{1}{T} \left| \overline{\pi}_{aT}^{P}(d) - \hat{\pi}_{aT}^{P} \right|. \tag{12}$$

Bai, Goldstein, and Yang (2019) find that estimates of leverage for C-rated firms using book values of debt are overstated and this may bias the estimate of the default boundary. For this reason we exclude the major rating category C in the estimation of the default boundary in equation (12) for all models.

Moody's provide an annual report with historical cumulative default rates and these are extensively used in the academic literature. The default rates are based on a long history of default experience for firms in different industries and different regions of the world. In Appendix B we use Moody's default database to calculate historical default rates for U.S. industrial firms and find them to be economically and statistically significantly different from those published by Moody's for global firms. We therefore use historical default rates for U.S. industrial firms calculated using default data from the period 1970–2017; given in Table A3 ('US industrial firms, equal-weight').

Table 4 shows the estimated default boundaries. The estimate of 0.8588 in the BC-0G model is similar to the estimate of 0.8944 in Feldhütter and Schaefer (2018), while the default boundary is lower in the BC model because the future level of debt is higher than in the BC-0G model. The SD model has a lower boundary than other models because there are two stochastic components that may narrow the gap between firm value and the default boundary: unexpected decreases in firm value and unexpected increases in debt level. Furthermore, we see that the models with firm-specific target leverage have a higher default boundary than those with constant target leverage, due to the average firm-specific leverage being lower than the constant leverage.

The estimate of 0.6269 in the SD-FL model is fairly close to the empirical estimate of 0.66 in Davydenko (2013). The estimate may seem high relative to the historical recovery rate of 33.48% for bonds, but the wedge can be explained by a higher recovery on bank debt and bankruptcy costs. Davydenko, Strebulaev, and Zhao (2012) estimate that default is partially a surprise and the market value of assets at default falls by half of the total default costs. Using Glover (2016)'s estimated default costs of 45% of firm value, the post-default firm value is 0.6269*(1-0.50*0.45)=0.49. This is close to Schwert (2019)'s estimate of 0.51 of post-default firm-level recovery and he reports that the 0.51 is decomposed into a 0.84 recovery on bank loans and 0.32 recovery on bonds. Thus, the estimated default boundary of 0.6269 in the SD model is consistent with empirical estimates of the default boundary in the literature and with a historical bond recovery rate of 0.3348 used later in bond pricing. In contrast, the estimated default boundary in the BC-0G and CDG models appear too high.

6 Pricing of corporate bonds

In this section we compare the structural models in their ability to price corporate bonds. We first investigate the time series variation of monthly average spreads and then proceed with spread predictions of individual bonds and finally end with average spreads prediction over the whole sample period.

6.1 Average monthly spreads

We investigate the models' ability to capture the time series variation of spreads by calculating time series of monthly average actual and model-implied spreads. Specifically, in each month t we calculate the average actual and model-implied spread (in basis points), s_t^a and s_t^M respectively, to obtain monthly time series for months $t = 1, ..., T$. Table 5 shows the average absolute log-pricing error

$$\frac{1}{T}\sum_{t=1}^{T}\Big|\log(s_t^a + 1) - \log(s_t^M + 1)\Big| \tag{13}$$

for a given rating class and range of bond maturity. We focus on log-prices instead of prices to reduce the influence of the 2008-2009 financial crisis. The table has three maturity ranges, 0–3 years (Short), 3–10 years (Medium), and 10–20 years (Long). Across maturities, the SD-FL and SD models have the smallest average pricing errors for both investment grade and speculative grade bonds. For example, the SD-FL model has average errors of 0.49 (investment grade) and 0.44 (speculative grade), lower than the other models. In contrast, the BC model does worst among the models with average errors of 0.96 and 0.63, respectively. The differences in model performance is predominantly due to large differences in the models' ability to capture short-term spreads. For example, the SD and SD-FL models have average errors of 0.70 and 0.61, respectively, for investment grade while the remaining models have errors in the range 1.09-1.89.

Figure 7 shows the monthly time series of short-term investment grade log-spreads. The figure shows why the SD and SD-FL models do better than the other models in capturing short-term spreads: the SD and SD-FL model spreads are close to actual spreads (apart from the period 2010-2014), while the other models predict too low spreads for most of the sample period. While it is not immediately clear from the figure, the predictions of BG-0G and CDG models flip sharply during the 2008-2009 financial crisis and the model spreads become much too high. For example, the average actual spread during (outside) 2008-2009 is 382bps (48bps), while it is 660bps (16bps) in the BG-0G model and 288bps (33bps) in the SD-FL model. While the average short-term investment grade spread in the BG-0G of 98bps matches the average actual spread of 91bps well, this is achieved by predicting too low spreads in normal times and too high spreads during the financial crisis. The predictions in the CDG and CDG-FL models are similar to those in the BC-0G model. In contrast, the average spread in the SD-FL model is lower (65bps) than in the BC-0G model but closer to the actual spread in both normal times and during the crisis.

Why are (short-term) spreads in the SD model higher in normal times and lower in the crisis than those in the BC-0G and CDG models? To see this we interpret the findings through the lens of the standard Merton model. Although the Merton model only allows for constant debt and default at bond maturity, the underlying mechanism is the same and the Merton model has a simple closed-form solution allowing us to understand the intuition clearly. In the Merton model the probability of default is given as $N(-D2D)$ where N is the cumulative normal distribution function and D2D (distance-to-default) is given as $D2D = \frac{ln(\frac{V_0}{dK_0})+(\mu-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$. To simplify further, we look at one-year spreads, i.e. $T = 1$, and ignore the drift term since this only has a modest impact on $D2D$ at short horizons. In this case we have

$$D2D \approx \frac{ln(\frac{V_0}{K_0}) - ln(d)}{\sigma}. \tag{14}$$

The default boundary d in each model is calibrated to historical default rates, represented by historical average distance-to-default ($\overline{D2D}$) and average log-leverage ($\overline{ln(L)}$). Ignoring convexity effects, the default boundary is estimated as

$$ln(d) = \overline{ln(L)} - \sigma * \overline{D2D}. \tag{15}$$

Inserting the default boundary estimate from (15) into equation (16) we have

$$D2D - \overline{D2D} \approx \frac{ln(L_0) - \overline{ln(L)}}{\sigma}. \tag{16}$$

Leverage volatility in the BC and CDG models is equal to asset volatility, σ_V, while it is higher in the SD model, $\sigma_{SD} > \sigma_V$, due to additional volatility coming from debt (see equation (24)). Combining this with equation (16) we see that

$$(D2D_{SD} - \overline{D2D}) \approx \frac{\sigma_V}{\sigma_{SD}}(D2D_{nonSD} - \overline{D2D}). \tag{17}$$

Equation (17) shows that changes in $D2D$ are less extreme in the SD model compared to the other models: in a calm period (where $D2D$ is higher than average), we have that $D2D_{SD} < D2D_{nonSD}$ while in a distressed period (where $D2D$ is lower than average), we have that $D2D_{SD} > D2D_{nonSD}$. Empirically, we find that $\frac{\sigma_V}{\sigma_{SD}} = 0.60$ so the increased stability of $D2D$ in the SD model is substantial.[19]

6.2 Individual spreads

We now compare model pricing at the individual transaction level. For a given rating and maturity bracket, we find all actual bond spread observations in the sample period that have this rating and maturity, $s_1^a, ..., s_N^a$. The corresponding model spreads are $s_1^M, ..., s_N^M$ and

[19]Since $\sigma_{SD} = \sqrt{\sigma_k^2 + \sigma_V^2 - 2\rho\sigma_k\sigma_V}$, $\rho = -0.6617$, and $\sigma_k = 0.8325\sigma_V$ we have that $\sigma_{SD} = 1.6718\sigma_V$.

Table 6 shows the average absolute log-pricing errors

$$\frac{1}{N}\sum_{i=1}^{N}\Big|\log(s_i^a+1)-\log(s_i^M+1)\Big|. \tag{18}$$

The table shows that the average pricing errors are smallest for the SD model for both investment grade and speculative grade bonds. The improved pricing of the SD model is due to more accurate pricing of short-term bonds. For example, the range of average absolute pricing errors for short-term bonds in the SD models is 1.58-1.73 (1.15-1.30) for investment (speculative) grade bonds while the range is 1.78-2.35 (1.42-1.60) for the four remaining models.

As discussed in the previous section, stochastic debt leads to higher spreads in calm periods and smaller spreads in distressed periods. This result holds in the cross-section as well, i.e. safe (risky) firms have higher (lower) spreads in the SD models compared to the models without stochastic debt. To illustrate this, Figure 8 shows a QQ plot of predicted credit spreads against actual spreads for short-term investment grade bonds. The two top graphs show that the BC and CDG models have QQ-plots substantially above the diagonal line. This shows that the models predict too low spreads for a substantial fraction of observations. The bottom graph shows that the SD models have QQ plots above the diagonal line as well, but the lines are closer to the diagonal line. This shows that although the SD models predict too low spreads in some cases, the distribution of spreads align more with the actual distribution compared with that of the other models.

We note that the individual pricing errors in Table 6 are generally large and significantly larger than the monthly pricing errors in Table 5. One potential explanation is that firm-time parameters are estimated with significant noise and this noise is particularly important when pricing at transaction level while the noise "washes out" when pricing portfolios.

6.3 Average spreads

Table 7 shows the average actual and model-implied spreads. Focussing on investment grade spreads, the table shows that the only model clearly failing to capture the average slope of the term structure of spreads is the BC model: the average model spread is 39bps (173bps) for short-term (long-term) spreads while the average actual spread is 77bps (121bps). The reason for this is that although the BC model predicts an average increase in debt consistent with actual firm debt dynamics, the model predicts a too strong increase in long-term debt for highly leveraged firms as Figure 5 shows. Since spreads of bonds issued by risky firms - in the cross section as well as the time series - contribute most to the average spread, the BC model predicts too high average long-term investment grade spreads. Likely, the failure to capture the average term structure is an important reason for the surprising absence of the model in empirical implementations of structural models in the literature.

The BC-0G, and SD-FL models have the best fit to average investment grade spreads across bond maturities. For example, their predictions for the average long-term spread are in the range 110-119bps, close to the actual average spread of 121bps. In unreported results we find that the SD-FL model has median spreads closer to actual median spreads: the median long-term investment grade spread is 81bps (52bps) in the SD-FL (BC-0G) model while the actual median spread is 89bps. This is not surprising since we showed previously that the SD model has a distribution of model spreads closer to the actual distribution.

Pricing differences between the models are smaller for speculative grade than for investment grade. All models underpredict long-term speculative grade spreads: the average actual long-term speculative grade spread is 470bps wile it is 270-366bps in the models.

6.4 Alternative estimation of the default boundary

We estimate the default boundary for each model by calibrating model-implied default probabilities to historical default rates across rating and horizon according to equation (12). In the calibration we exclude C-rated firms because Bai, Goldstein, and Yang (2020) (BGY) find that assuming that the market value of debt is equal to the book value of debt when calculating the leverage ratio may be problematic for C-rated firms.

BGY suggest an alternative approach to what we propose here. For each rating class, they calculate the average market-to-book ratio of bonds and suggest, when calculating leverage, using this market-to-book ratio as a proxy for the market-to-book ratio of total debt for all firms with the corresponding rating. With this adjustment they include C-rated firms in the calibration in equation (12).

BGY make an important point in highlighting the wedge between book values and market values of debt, but their adjustment is potentially biased because they only use bond data when adjusting for the wedge. Debt consists of both bank loans and corporate bonds, and since bank loans are senior to bonds, recovery rates on bank loans are higher. This in turn implies that loan prices are typically closer to book values than bond prices are, and thus BGY are adjusting “too much” by relying on bond prices. Bretscher, Feldhütter, Kane, and Schmid (2020) (BFKS) investigate market values of debt using a data set of both bond and bank debt transactions and find that the bias in the BGY adjustment is severe. For example, BGY find the average market-to-book ratio for C-rated firms to be 0.72 when using only bond data while BFKS find it to be 0.95 when using bond and bank debt data.[20]

In our estimation we minimize the potential bias by excluding the rating category C when calibrating models in equation (12). At the loss of some statistical accuracy we avoid making any assumption about the market value of debt of C-rated companies. To see how sensitive our estimation results are to alternative assumptions, Table 8 shows default boundary estimates and average spreads for different ratings when we follow the approach in BGY (include C-rated firms in the default boundary calibration and use the adjustment factors in BGY) and BFKS (include C-rated firms in the default boundary calibration and use the adjustment factors in BFKS).

Feldhütter and Schaefer (2018) (FS), BGY, and BFKS use the BC-0G model and the table shows that when applying the BGY adjustment there is a strong impact on the estimation results compared to making no adjustment: the estimated default boundary drops from 0.86 to 0.71 and the average BBB spread, for example, drops from 145bps to 81bps. This implies that if we adjust the book value of debt as suggested in BGY and include C-rated firms in equation (12) rather than making no adjustment, results change dramatically. As discussed above BGY adjust ”too much” and ’BFKS adjustment’ in the table shows results when improving the adjustment by taking into account bank debt. We see that the adjustment

[20]The ratio of market value to book value of bonds reported in BGY is 1.10, 1.07, 1.03, 1.02, 1.01, 1.00, and 0.72 for AAA, AA, A, BBB, BB, B, C, respectively. BFKS compute the average ratio of market value to book value of total debt to be 1.01, 1.02, 1.04, 1.05, 1.01, 0.99, and 0.95 for AAA, AA, A, BBB, BB, B, C, respectively. Their samples are not the same and therefore the number are not directly comparable. For example, BFGS find a ratio of 0.82 when they calculate the BGY adjustment factor for C-rated firms in their data sample. See also Shi (2019) and Huang, Nozawa, and Shi (2020) for further estimates of ratios.

has a much smaller effect on the estimation results and, in fact, are mostly economically small. For example, the default boundary estimate is 0.87 instead of 0.86 in the BC-0G model and average spreads are similar with the exception of C-rated firms. For C-rated firms, spreads are raised substantially and more in line with actual spreads. Results are similar for the CDG and SD-FL models: the impact of adjusting for the market value of debt is much smaller when taking into account bank debt as theory requires and, except for C-rated firms, the impact on spreads modest. Since our main focus is on short-term investment grade spreads, the table shows that making market value adjustments to debt as suggested by BGY has no important impact on our results once the adjustment takes into account bank debt.

There is a substantial literature finding that standard structural models imply investment grade spreads that are substantially below actual credit spreads, the so-called credit spread puzzle. This literature uses the historical default rate for a single rating and maturity as a proxy for the default probability and FS document that the statistical power when doing so is low. Instead, FS propose using the statistical more precise equation (12) (including C-rated firms) when fitting to historical default rates and find that the credit spread puzzle disappears. In contrast, BGY find that the puzzle reappears when using their proposed adjustment to leverage. Both papers focus on the BC-0G model. Since we have a range of models and furthermore apply the more accurate BFKS adjustment factors, we can provide new and more robust evidence as to the existence of a puzzle.

FS and BGY investigate the BC-0G model and Table 8 shows that the average BBB-spread – which has been the focus of much of the literature – is 145bps in our main results, called 'Benchmark', close to the actual spread of 141bps. There is a significant effect on average spreads of using the adjustment suggested by BGY (when including the C category) in the estimation: the average BBB spread is 81bps and the difference to the actual spread is statistically highly significant. These results are close to those reported in BGY. However, when applying BFKS's more accurate adjustment factors, the average BBB spread is 130bps and the results are again close to the benchmark results as also pointed out by BFKS. Turning to the CDG and SD-FL models we see a similar pattern: the BGY adjustment leads to lower spreads compared to the benchmark case, but using the BFKS adjustment returns spreads to close to those of the benchmark case and close to actual spreads. The only exception is C-rated bonds where the BFKS adjustment brings model spreads close to actual spreads and this is the case for all three models. We conclude that the finding in FS that standard structural models can match investment grade spreads is robust across a range of models. Furthermore, adjusting leverage ratios to account for a potential wedge between market and book values of debt allows the models to match speculative grade spreads as well. Thus, the is no credit spread puzzle for neither investment grade nor speculative grade spreads.

7 Conclusion

We investigate how to specify the default boundary – as measured by the level of debt – in structural models of credit risk. Empirically, the level of debt increases over time and the future debt level is negatively related to current leverage. Furthermore, we document that contemporaneously with a negative equity shock, a firm increases debt relatively more, but after the equity shock, the firm decreases debt relatively more to an overall lower level.

We incorporate these debt dynamics into the dynamics of the default boundary into an otherwise standard structural model (where firm value follows a Geometric Brownian Motion and the Sharpe Ratio is constant). Compared to existing models (the Black-Cox model with or without a future increase in debt and the stationary leverage model of Collin-Dufresne and Goldstein (2001)), the model has more accurate pricing predictions, particularly for short-maturity investment grade bonds.

A Analytical results

All models assume that firm-value dynamics follows a geometric Brownian Motion (see equation (3)):

$$\frac{dV_t}{V_t} = (\mu - \delta)dt + \sigma dW_t^P, \tag{19}$$

where μ is the expected return on the firm's assets, δ is the payout ratio, and σ is the asset return volatility. The dynamics for the log of firm value, $v_t = \log V_t$, is

$$dv_t = (\mu - \delta - \frac{\sigma^2}{2})dt + \sigma dW_t^P. \tag{20}$$

A.1 Default probabilities

In the Black-Cox model the debt level is given as $K(t) = K_0 e^{\gamma t}$ and the cumulative default probability at time t is (see Bao (2009))

$$\begin{aligned} \pi^P(dL_0, \Theta^P, t) &= N\Big[-\Big(\frac{(-\log(dL_0) + a_0 t}{\sigma\sqrt{t}}\Big)\Big] + \exp\Big(\frac{2\log(dL_0)a_0}{\sigma^2}\Big)N\Big[\frac{\log(dL_0) + a_0 t}{\sigma\sqrt{t}}\Big] \\ a_0 &= \mu - \delta - \gamma - \frac{\sigma^2}{2} \end{aligned} \tag{21}$$

where $L_0 = \frac{K_0}{V_0}$ is the current leverage and $\Theta^P = (\mu, \sigma, \delta, \gamma)$. The risk-neutral default probability, π^Q, is obtained by replacing μ with r in equation (21). The default probability in the constant boundary model is given by setting $\gamma = 0$ in (21).

In the stochastic debt model the dynamics of the log-debt level, k_t, is (see equation (10))

$$dk_t = \lambda(\nu - l_t)dt + \sigma_k dW_{k,t} \tag{22}$$

If we assume that the bond Sharpe Ratio is θ, then l_t follows the following risk-neutral dynamics[21]

$$dl_t = \lambda(\bar{l}^Q - l_t)dt + \sigma_k dW_{k,t} - \sigma dW_t^Q, \tag{23}$$

where $\bar{l}^Q = \nu - \frac{r + (\sigma_l - \sigma_V)\theta - \delta - \frac{\sigma^2}{2}}{\lambda}$. It is convenient to write the dynamics as

$$dl_t = \lambda(\bar{l}^Q - l_t)dt + \sigma_l dW_{l,t} \tag{24}$$

[21] It is straightforward to show that the dynamics in the SD model of l_t under P and Q is the same as the dynamics in the CDG model with σ replaced by σ_l and ν replaced by ν^* where $\nu^* = \nu + \frac{\sigma_V^2}{2\lambda} - \frac{\sigma_l^2}{2\lambda} + \frac{\sigma_l\theta^V}{\lambda} - \frac{\sigma_V\theta^V}{\lambda}$. Assuming a bond Sharpe ratio of θ in the CDG model then gives rise to the risk-neutral dynamics in equation (23). Alternatively, one can assume that the CAPM holds and derive the result using the Feynman-Kac theorem.

where $\sigma_l = \sqrt{\sigma_k^2 + \sigma^2 - 2\rho\sigma_k\sigma}$. Since the default time is $\tau = \inf\{t|0 \leq l_t + log(d)\}$, we define $\tilde{l}_t = l_t + log(d)$ which has the dynamics

$$\tilde{l}_t = \lambda(\bar{\bar{l}}^Q - \tilde{l}_t)dt + \sigma_l dW_{l,t}, \tag{25}$$

where $\bar{\bar{l}}^Q = \bar{l}^Q + \log(d)$. The default time is the first time $\tilde{l}$ hits 0.

Log-leverage in equation ((25)) follows an Ornstein-Uhlenbeck (OU) process and the default time is therefore the first hitting time (FHT) of an OU process. There are no closed-form solutions for the distribution of the FHT of an OU process in the general case and we use numerical methods.

A.2 Expected value of debt level

The expected debt level in the Merton model is constant while it is deterministically increasing in the Black-Cox model.

To calculate the expected debt level in the stochastic debt model, we can write the dynamics for leverage under the natural measure as:

$$dl_t = \lambda(\bar{l}^P - l_t)dt + \sigma_l dW_{l,t} \tag{26}$$

where W^P is the Brownian motion driving firm value, W_i is independent of W^P, and $\bar{l}^P = \nu - \frac{\mu - \delta - \frac{\sigma^2}{2}}{\lambda}$.

We have that both v_t and l_t are normally distributed,

$$\begin{aligned} v_t &= v_0 + (\mu - \delta - \frac{\sigma^2}{2})t + \sigma W_{v,t} && (27) \\ l_t &= \bar{l} + e^{\lambda t}(l_0 - \bar{l}) + (\sigma_k \rho - \sigma)e^{\lambda t}\int_0^t e^{-\lambda s}dW_{v,s} + \sigma_k\sqrt{1-\rho^2}e^{\lambda t}\int_0^t e^{-\lambda s}dW_{i,s} && (28) \end{aligned}$$

This immediately gives

$$E(k_t) = v_0 + (\mu - \delta - \frac{\sigma^2}{2})t + \bar{l} + e^{\lambda t}(l_0 - \bar{l}). \tag{29}$$

The covariance is

$$
\begin{aligned}
Cov(l_t, v_u) &= Cov\Big((\sigma_k\rho - \sigma)e^{\lambda t}\int_0^t e^{-\lambda s}dW_{v,s} + \sigma_k\sqrt{1-\rho^2}e^{\lambda t}\int_0^t e^{-\lambda s}dW_{i,s}, \sigma W_{v,u}\Big) && (30)\\
&= (\sigma_k\rho - \sigma)e^{\lambda t}\sigma E\Big(\int_0^t e^{-\lambda s}dW_{v,s} \times W_{v,u}\Big) && (31)\\
&= (\sigma_k\rho - \sigma)e^{\lambda t}\sigma E\Big(\int_0^t e^{-\lambda s}dW_{v,s} \times \int_0^u dW_{v,s}\Big) && (32)\\
&= (\sigma_k\rho - \sigma)e^{\lambda t}\sigma E\Big(\int_0^{\min(t,u)} e^{-\lambda s}ds) && (33)\\
&= (\sigma_k\rho - \sigma)e^{\lambda t}\sigma\frac{1}{\lambda}(1 - e^{-\lambda\min(t,u)}) && (34)
\end{aligned}
$$

where the Itô isometry is used. Their correlation is

$$
\begin{aligned}
\rho_{l_t,v_u} = Corr(l_t, v_u) &= \frac{(\sigma_k\rho - \sigma)e^{\lambda t}\sigma\frac{1}{\lambda}(1 - e^{-\lambda\min(t,u)})}{\sqrt{\sigma^2 u}\sigma_{l_t}} && (35)\\
&= \frac{(\sigma_k\rho - \sigma)e^{\lambda t}(1 - e^{-\lambda\min(t,u)})}{\lambda\sigma_{l_t}\sqrt{u}} && (36)
\end{aligned}
$$

where we have defined $\sigma_{l_t} = \sqrt{(\sigma_k^2 + \sigma^2 - 2\sigma\sigma_k\rho)\frac{e^{2\lambda t}-1}{2\lambda}}$

According to Azzalini and Valle (1996) p. 716-717 we have that $\frac{l_t - E_0(l_t)}{\sigma_{l_t}}$ given $\frac{v_u - E_0(v_u)}{\sigma_{v_u}} > 0$ is skew-normal distributed with mean $\sqrt{\frac{2}{\pi}}\rho_{l_t,v_u}$ and therefore

$$
E[l_t|v_u > E_0(v_u)] = \sqrt{\frac{2}{\pi}}\frac{(\sigma_k\rho - \sigma)e^{\lambda t}(1 - e^{-\lambda\min(t,u)})}{\lambda\sqrt{u}} + \bar{l} + e^{\lambda t}(l_0 - \bar{l}). \quad (37)
$$

Now

$$
E[v_t|v_u > E_0(v_u)] = v_0 + (\mu - \delta - \frac{\sigma^2}{2})t + \sigma E[W_{v,t}|W_{v,u} > 0] \quad (38)
$$

and using the results in Azzalini and Valle (1996) we have $E[W_{v,t}|W_{v,u} > 0] = \sqrt{\frac{2}{u\pi}}\min(t,u)$ so

$$
\begin{aligned}
E[k_t|v_u > E_0(v_u)] &= \sqrt{\frac{2}{\pi}}\frac{(\sigma_k\rho - \sigma)e^{\lambda t}(1 - e^{-\lambda\min(t,u)})}{\lambda\sqrt{u}} && (39)\\
&\quad + \bar{l} + e^{\lambda t}(l_0 - \bar{l}) + v_0 + (\mu - \delta - \frac{\sigma^2}{2})t + \sigma\sqrt{\frac{2}{u\pi}}\min(t,u). && (40)
\end{aligned}
$$

Likewise

$$E[k_t|v_u < E_0(v_u)] = -\sqrt{\frac{2}{\pi}}\frac{(\sigma_k\rho-\sigma)e^{\lambda t}(1-e^{-\lambda\min(t,u)})}{\lambda\sqrt{u}} \tag{41}$$

$$+\bar{l}+e^{\lambda t}(l_0-\bar{l})+v_0+(\mu-\delta-\frac{\sigma^2}{2})t-\sigma\sqrt{\frac{2}{u\pi}}\min(t,u). \tag{42}$$

so

$$E[k_t|v_u > E_0(v_u)] - E[k_t|v_u < E_0(v_u)] = \sqrt{\frac{8}{\pi u}}\Big[\frac{(\sigma_k\rho-\sigma)e^{\lambda t}(1-e^{-\lambda\min(t,u)})}{\lambda}+\sigma\min(t,u)\Big]. \tag{43}$$

Since v_u is normally distributed the mean and the median is the same and therefore the above formula is the same if we condition on v_u being larger than the median instead of the mean. Also, there is a monotone relation between the value of equity and the value of the firm, and the above formula therefore holds if we condition on equity value being larger than the median instead of firm value being larger than the median.

The same formulas hold for the stationary leverage model with $\sigma_v = \rho = 0$.

B Default rate calculations

Moody's provide an annual report with historical cumulative default rates and these are extensively used in the academic literature as estimates of default probabilities. The default rates are based on a long history of default experience for firms in different industries and different regions of the world.

A number of studies find that ratings across industries and regions are not comparable: Cornaggia, Cornaggia, and Hund (2017) find that default rates of financial institutions are significantly different from default rates of similarly-rated non-financial institutions, Cantor and Falkenstein (2001) find default rates of speculative-grade utility firms are significantly different from default rates of similarly-rated non-utility firms, while Cantor, Stumpp, Madelain, and Bodard (2004) find that ratings are more accurate for European firms than for North American firms.

As in most studies of structural models of credit risk we focus on U.S. industrial firms. To compare apples with apples we therefore use Moody's default database to calculate historical default rates for the subset of U.S. industrial firms in the Moody's database. Hamilton and Cantor (2006) discuss how Moody's calculate default rates (Moody's approach is based on the methodology in Altman (1989) and Asquith, Mullins, and Wolff (1989)) and we review their methodology in Appendix B.1.

In Appendix B.2 we detail our calculation of default rates for U.S. industrial firms and compare our results with those published by Moody's for all global firms. We find that default rates for U.S. industrial firms are economically different from Moody's published default rates for all rated firms. In Appendix B.3 we show that the differences in default rates are also statistically significant.

B.1 Moody's default rate calculations

Assume that there is a cohort of issuers formed on date y holding rating z. The number of firms in the cohort during a future time period is $n_y^z(t)$ where t is the number of periods from the initial forming date (time periods are measured in months in the main text). In each period there are three possible mutually exclusive end-of-period outcomes for an issuer: default, survival, and rating withdrawal. The number of defaults during period t is $x_y^z(t)$, the number of withdrawals is $w_y^z(t)$, and the number of issuers during period t is defined as

$$n_y^z(t) = n_y^z(0) - \sum_{i=1}^{t-1} x_y^z(i) - \sum_{i=1}^{t-1} w_y^z(i) - \frac{1}{2} w_y^z(t). \tag{44}$$

The marginal default rate during time period t is

$$d_y^z(t) = \frac{x_y^z(t)}{n_y^z(t)} \tag{45}$$

and the cumulative default rate for investment horizons of length T is

$$D_y^z(T) = 1 - \prod_{t=1}^{T} \Big[1 - d_y^z(t)\Big]. \tag{46}$$

The average cumulative default rate is

$$\overline{D}^z(T) = 1 - \prod_{t=1}^{T} \left[1 - \overline{d}^z(t)\right] \tag{47}$$

where $\overline{d}^z(t)$ is the average marginal default rate[22].

For a number of cohort dates y in a historical data set Y, Moody's calculate the average marginal default rate as a weighted average, where each period's marginal default rate is weighted by the relative size of the cohort

$$\overline{d}^z(t) = \frac{\sum\limits_{y \in Y} x_y^z(t)}{\sum\limits_{y \in Y} n_y^z(t)}. \tag{48}$$

We label default rates based on equation (48) for *cohort-weighted* default rates. In the presence of macroeconomic risk as modelled in Feldhütter and Schaefer (2018) it is more robust to use *equal-weighted* default rates where the average marginal default rate is calculated as

$$\overline{d}^z(t) = \frac{1}{N_Y} \sum_{y \in Y} \frac{x_y^z(t)}{n_y^z(t)} \tag{49}$$

where N_Y is the number of cohorts in the historical dataset Y.

B.2 Calculating default rates for U.S. industrial firms

Moody's default database appears to have a more extensive coverage of firms in the last 50 years compared to the previous 50 years. Specifically, there are 9,055 firms with a rating at some point in the period 1919–1969 while there are 27,549 in the period 1970–2018. We therefore restrict our calculation of default rates to start from January 1, 1970.

We calculate historical default rates for industrial firms but, as a check on our methodology, we first replicate Moody's default rates for all firms. Table A3 shows Moody's (2018a) reported historical default rates 1970–2017 and, in row 2, default rates for all firms calculated

[22]Note that this calculation assumes that marginal default rates are independent.

using Moody's methodology and their default database for the same sample period (January 1, 1970 to January 1, 2018) as in Moody's (2018a). The calculated default rates are close, but not identical, to Moody's reported default rates. For example, the 10-year BBB default rate, a focal point in the academic literature, is calculated to be 3.83% while Moody's report 3.75%. We do not expect to replicate Moody's rates exactly because Moody's (2018b) note that "you will not be able to replicate the exhibits exactly, as the researchers have access to non-public information that is not included in the database".

When we restrict the sample to US industrial firms, historical default rates change quite dramatically as 'US industrial firms, cohort weight' (row 3 in Table A3) shows. The BBB 10-year default rate, for example, is estimated to be 5.74% which is 53% higher than Moody's estimate. Thus, there is a substantial effect of restricting the sample to U.S. industrial firms. We show in Appendix B.3 that the difference in default rates when using all firms and when using US industrial firms is statistically significant and therefore it is important to restrict the sample to correspond to firms used in our empirical analysis.[23]

Moody's calculate average default rates by using a cohort-weighted average of default rates. This leads to an uneven weighting across time. For example, default rates for AAA (B) during the decade 1970–1979 are weighted 4.5 times higher (23 times lower) than default rates during the most recent decade 2008-2017. In the presence of macroeconomic risk, it is preferable to have an even weighting across time, and we therefore equally-weight default rates. Table A3 shows that there is a moderate effect on default rates of equal weighting. The 10-year BBB default rate increases from 5.74% to 6.43%. However, there is no clear pattern in the direction that default rates change generally.

B.3 Are default rates of US industrial firms different from those of all firms

In this section we calculate the statistical significance of the difference in default rates calculated using US industrial firms and firms that are not US industrials using default

[23]It may be surprising that the difference is significant given that Feldhütter and Schaefer (2018) show that default rates have large confidence bands. However, the difference in default rates for two samples exposed to the same macroeconomic shocks is more precisely estimated than either default rate separately.

data from 1970–2017. We do so by calculating the distribution of the difference under the assumption that both sets of firms have the same ex ante default probability.

Specifically, for a given rating r and horizon h (in years 1,...,20), we record the number of firms in the January 1970, January 1971, ..., January 2017+1-h US industrial cohorts, $n^1_{1970}, n^1_{1971}, ..., n^1_{1977+1-h}$ (sample 1) and the corresponding number in cohorts of the remaining firms, $n^2_{1970}, n^2_{1971}, ..., n^2_{1977+1-h}$ (sample 2). We calculate the historical equal-weighted default rate, $\hat{p}_{r,h}$ as in the previous Section B.1 for all firms by combining the cohorts n^1_y and n^2_y (combined sample).

In year 1, corresponding to cohort 1970, we have $n^1_{1970} + n^2_{1970}$ firms, where firm i's value under the natural measure follows a GBM,

$$\frac{dV^i_t}{V^i_t} = (\mu - \delta)dt + \sigma dW^P_{it}. \tag{50}$$

We assume every firm has one h-year bond outstanding, and a firm defaults if firm value is below face value at bond maturity, $V^i_h \leq F$. Using the properties of a Geometric Brownian Motion, the default probability is

$$p = P(W^P_{iT} - W^P_{i0} \leq c) \tag{51}$$

where $c = \frac{\log(F/V_0) - (\mu - \delta - \frac{1}{2}\sigma^2)T)}{\sigma}$. This implies that the unconditional default probability is $N(\frac{c}{\sqrt{T}})$ where N is the cumulative normal distribution. For a given default probability $\hat{p}_{r,h}$ we can always find c such that equation (51) holds, so in the following we use $\hat{p}_{r,h}$ instead of the underlying Merton parameters that give rise to $\hat{p}_{r,h}$.

We introduce systematic risk by assuming that

$$W^P_{iT} = \sqrt{\rho} W_{sT} + \sqrt{1-\rho} W_{iT} \tag{52}$$

where W_i is a Wiener process specific to firm i, W_s is a Wiener process common to all firms, and ρ is the pairwise correlation between percentage firm value changes, which we set to $\rho = 0.2002$ following Feldhütter and Schaefer (2018). All the Wiener processes are independent. We simulate the realized default frequencies in the year 1-cohort separately for sample 1 and 2 by simulating one systematic, common for both samples, and $n^1_{1970} + n^2_{1970}$ idiosyncratic processes in equation (52).

In year 2 we form a cohort of $n^1_{1971} + n^2_{1971}$ new firms. The firms in year 2 have characteristics that are identical to those of the previous firms at the point they entered the index in year 1. We calculate the realized h-year default frequency of the year 2-cohort as we did for the year 1-cohort. Crucially, the common shock for years 1-9 for the year 2-cohort is the same as the common shock for years 2-10 for firms in the year 1-cohort. We repeat the same process for $48 - h$ years and calculate the overall realized cumulative 10-year default frequency for the two samples, $\tilde{p}^1_{r,h}$ and $\tilde{p}^2_{r,h}$, by taking an average of the default frequencies across the $48 - h$ cohorts, and compute the difference $\tilde{d}^{s_1}_{r,h} = \tilde{p}^1_{r,h} - \tilde{p}^2_{r,h}$. Finally, we repeat this entire simulation 100,000 times, get $\tilde{d}^{s_1}_{r,h}, \tilde{d}^{s_2}_{r,h}, ..., \tilde{d}^{s_{100,000}}_{r,h}$. A historical difference is significant at say the 5% level of this historical difference is smaller than the 2.5% quantile or larger than the 97.5% quantile in the simulated distribution of differences.

There are three approximations in this calculation. First, in the main text we use monthly cohorts while we use yearly cohorts in the simulation. Second, we assume all firms are replaced each year, while this is not so in the actual cohorts. These two approximations partially counterweight each other. Third, we do not use marginal default rates as above.

Table A4 shows the the statistical significance of the difference in default rates. We see that long-term default rates for risky firms (rated BBB or below) are higher than other firms and the difference is statistically highly significant. For example, the 20-year BBB default rate for US industrial firms is 15.83% which is 242% higher than that of other firms and the difference is significant at the 0.1% level.

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	Log-growth										Growth (in percent)		
Horizon (years)	1	2	3	4	5	6	7	8	9	10	1	5	10
Leverage 0-1	0.10	0.20	0.30	0.39	0.47	0.55	0.62	0.70	0.77	0.85	10	60	134
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)			
	[139902]	[123947]	[110745]	[99650]	[90186]	[81926]	[74596]	[68090]	[62119]	[56744]			
Leverage 0-0.2	0.23	0.45	0.64	0.80	0.94	1.06	1.16	1.27	1.37	1.46	26	155	331
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)			
	[64373]	[56869]	[50692]	[45495]	[41089]	[37256]	[33866]	[30914]	[28295]	[25980]			
Leverage 0.2-0.4	0.02	0.07	0.12	0.17	0.22	0.28	0.33	0.39	0.44	0.51	3	25	67
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)			
	[36087]	[32343]	[29186]	[26431]	[24026]	[21914]	[20048]	[18316]	[16757]	[15365]			
Leverage 0.4-0.6	-0.03	-0.04	-0.02	0.01	0.05	0.09	0.14	0.17	0.22	0.27	-3	5	31
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)			
	[22365]	[20047]	[17928]	[16133]	[14580]	[13234]	[12063]	[11093]	[10089]	[9188]			
Leverage 0.6-0.8	-0.08	-0.13	-0.15	-0.15	-0.12	-0.11	-0.08	-0.03	0.01	0.06	-7	-12	7
	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)			
	[12749]	[11089]	[9813]	[8797]	[7989]	[7268]	[6587]	[5941]	[5366]	[4795]			
Leverage 0.8-1	-0.17	-0.29	-0.36	-0.40	-0.38	-0.36	-0.33	-0.30	-0.26	-0.22	-16	-32	-20
	(0.01)	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)	(0.06)	(0.06)			
	[4328]	[3599]	[3126]	[2794]	[2502]	[2254]	[2032]	[1826]	[1612]	[1416]			

Table 1 *Future debt relative to current debt.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. The table shows the average log-ratio for different initial leverage ratios and future horizons. The final columns of the table shows for selected horizons the growth rate calculated as exp (log-growth rate) -1. Standard errors clustered at firm level is in parentheses and the number of observations in brackets. The data is from CRSP/Compustat and the sample period is 1965-2017.

horizon (years)	1	2	3	4	5	6	7	8	9	10
Leverage 0-0.2										
High equity return	0.22	0.43	0.60	0.80	0.97	1.10	1.21	1.35	1.44	1.57
Low equity return	0.36	0.63	0.78	0.81	0.89	1.00	1.08	1.17	1.28	1.42
Difference	−0.14**	−0.20**	−0.18**	−0.01	0.08*	0.09*	0.13**	0.17**	0.16**	0.14**
	(0.01)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)
Leverage 0.2-0.4										
High equity return	0.01	0.05	0.09	0.19	0.29	0.36	0.45	0.49	0.59	0.66
Low equity return	0.08	0.17	0.22	0.22	0.24	0.25	0.33	0.38	0.45	0.53
Difference	−0.07**	−0.13**	−0.13**	−0.03	0.05	0.11**	0.12**	0.11**	0.14**	0.13**
	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)
Leverage 0.4-0.6										
High equity return	-0.04	-0.06	-0.05	0.02	0.07	0.16	0.25	0.31	0.35	0.42
Low equity return	0.03	0.05	0.08	0.06	0.05	0.07	0.09	0.14	0.17	0.22
Difference	−0.07**	−0.11**	−0.13**	−0.04	0.02	0.09*	0.16**	0.17**	0.17**	0.20**
	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)	(0.05)
Leverage 0.6-0.8										
High equity return	-0.09	-0.17	-0.19	-0.14	-0.05	-0.01	0.01	0.05	0.06	0.16
Low equity return	-0.03	-0.05	-0.07	-0.12	-0.13	-0.13	-0.16	-0.10	-0.05	0.03
Difference	−0.06**	−0.11**	−0.12**	−0.01	0.09*	0.12*	0.17**	0.15*	0.11	0.12
	(0.01)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.06)	(0.06)	(0.07)	(0.07)
Leverage 0.8-1										
High equity return	-0.21	-0.30	-0.35	-0.34	-0.25	-0.22	-0.17	-0.05	-0.08	-0.08
Low equity return	-0.14	-0.23	-0.27	-0.34	-0.31	-0.40	-0.34	-0.26	-0.24	-0.23
Difference	−0.07**	−0.07	−0.07	−0.00	0.06	0.18	0.18	0.21	0.16	0.15
	(0.03)	(0.04)	(0.06)	(0.07)	(0.08)	(0.10)	(0.11)	(0.12)	(0.14)	(0.15)

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Table 2 *Future debt relative to current debt conditional on future three-year equity returns.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The table the average log-ratio for high and low future equity return firms as well as the difference. In parentheses are standard errors, clustered at the firm level, of the differences and '*' indicate significance at 95% level and '**' at 99% level. The data is from CRSP/Compustat and the sample period is 1965-2017.

0-20-year bond maturity

	AAA	AA	A	BBB	BB	B	C	all
Number of bonds	66	322	1486	1422	563	410	152	3129
Age	4.99	4.68	6.1	7.24	6.79	7.82	11.1	6.59
Coupon	5.6	6.22	6.99	7.61	7.58	7.46	7.73	7.18
Amount outstanding ($mm)	610	520	296	324	274	282	275	333
Time-to-maturity	6.87	6.45	6.69	7.11	6.78	6.16	7.90	6.81
Number of observations	2617	10673	36153	31178	11908	5679	2873	101081

0-3-year bond maturity

	AAA	AA	A	BBB	BB	B	C	all
Number of bonds	26	144	765	730	262	238	61	1884
Age	4.72	5.78	6.95	8.25	7.05	7.2	9.58	7.29
Coupon	3.94	4.73	5.82	6.62	6.92	6.63	7.44	6.15
Amount outstanding ($mm)	707	707	342	361	293	297	304	382
Time-to-maturity	1.59	1.43	1.45	1.39	1.51	1.47	1.61	1.44
Number of observations	513	2706	9668	8228	3072	1882	729	26798

3-7-year bond maturity

	AAA	AA	A	BBB	BB	B	C	all
Number of bonds	43	195	887	681	253	147	69	1924
Age	3.56	4.21	4.9	6.24	5.54	5.97	7.68	5.36
Coupon	5.85	6.23	7	7.85	7.61	7.72	7.27	7.23
Amount outstanding ($mm)	599	541	300	321	294	307	267	341
Time-to-maturity	4.73	4.82	4.81	4.79	4.86	4.75	4.84	4.81
Number of observations	1011	3812	12477	9120	4219	1923	824	33386

7-13-year bond maturity

	AAA	AA	A	BBB	BB	B	C	all
Number of bonds	29	123	474	482	182	85	54	1182
Age	4.51	3.59	5.5	6.48	6.8	10.2	14.2	6.13
Coupon	5.95	7.04	7.66	7.97	7.88	7.95	8.25	7.69
Amount outstanding ($mm)	674	378	263	318	239	245	219	303
Time-to-maturity	9.29	9.18	9.34	9.38	9.27	9.67	10.20	9.36
Number of observations	770	3044	8936	8776	2838	1153	618	26135

13-20-year bond maturity

	AAA	AA	A	BBB	BB	B	C	all
Number of bonds	9	40	188	235	93	56	41	499
Age	11.1	6.58	8.5	8.72	9.27	10.5	13.8	8.93
Coupon	6.64	7.62	8.05	8.14	8.19	8.12	8.12	8.04
Amount outstanding ($mm)	343	378	258	281	247	234	306	277
Time-to-maturity	16.21	16.73	16.63	16.66	16.48	16.52	16.00	16.59
Number of observations	323	1111	5072	5054	1779	721	702	14762

Table 3 *Bond summary statistics.* The sample consists of noncallable bonds with fixed coupons issued by industrial firms. This table shows summary statistics for the data set. The data sample cover the period 1988Q2-2018Q1. 'Number of bonds' is the number of bonds that appear (in a particular rating and maturity range) at some point in the sample period. 'Mean number of bonds pr month' is the average number of bonds that appear in a month. 'Mean number of quotes pr month' is the total number of quotes in the sample period divided by the number of months. For each quote we calculate the bond's time since issuance and 'Age' is the average time since issuance across all quotes. 'Coupon' is the average bond coupon across all quotes. 'Amount outstanding' is the average outstanding amount of a bond issue across all quotes. 'Time-to-maturity' is the average time until the bond matures across all quotes.

Model	Default boundary estimate ($\hat{d}$)
BC-0G	0.8588
BC	0.6700
CDG	0.8870
CDG-FL	0.8928
SD	0.5675
SD-FL	0.6269

Table 4 *Estimates of the default boundary.* For each structural model we estimate the default boundary – the fraction of the total face value of debt at which the firm defaults – by minimizing the distances between average model-implied default probabilities and historical average default rates for a cross-section of horizons 1,...,20 years and ratings AAA, AA, A, BBB, BB, B. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. The historical average default rates are based on defaults of U.S. industrial firms in the period 1970–2017.

		Average	Short	Medium	Long
Inv	BC-0G	0.64	1.27	0.38	0.27
	BC	0.96	1.89	0.40	0.60
	CDG	0.60	1.09	0.35	0.37
	CDG-FL	0.71	1.19	0.43	0.50
	SD	0.63	0.70	0.62	0.56
	SD-FL	0.49	0.61	0.54	0.32
Spec	BC-0G	0.50	0.64	0.42	0.44
	BC	0.63	1.16	0.44	0.30
	CDG	0.52	0.62	0.44	0.50
	CDG-FL	0.55	0.64	0.55	0.46
	SD	0.50	0.60	0.40	0.50
	SD-FL	0.44	0.49	0.44	0.39

Table 5 *Log-pricing errors of monthly average credit spreads.* This table shows how well structural models match average monthly credit spreads. For a given rating r and maturity m, we find all bonds at the end of a given month t that have this rating and maturity, calculate the average actual credit spread (in basis points) to the swap rate, s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of monthly average model credit spread (in basis points) $s^M_{rm1}, ..., s^M_{rmT}$. This table shows the average absolute log-pricing error $1/T \sum_{t=1}^{T} \left| \log(s^a_{rmt} + 1) - \log(s^M_{rmt} + 1) \right|$. 'Short' includes bond maturities in the range 0-3 years, 'Medium' 3-10 years, and 'Long' 10-20 years. 'BC-0G' refers to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. The sample period for 'Short' is 2002:07-2018:03 while it is 1988:03-2018:03 for 'Medium' and 'Long'.

		Average	Short	Medium	Long
Inv	BC-0G	1.96	2.92	1.87	1.10
	BC	1.94	3.07	1.82	0.93
	CDG	1.78	2.80	1.61	0.91
	CDG-FL	2.35	2.92	2.29	1.84
	SD	1.58	2.50	1.29	0.95
	SD-FL	1.73	2.54	1.54	1.10
Spec	BC-0G	1.48	2.60	1.07	0.78
	BC	1.60	3.12	1.08	0.62
	CDG	1.42	2.36	1.03	0.87
	CDG-FL	1.55	2.46	1.26	0.93
	SD	1.30	2.02	0.94	0.92
	SD-FL	1.15	1.89	0.86	0.70

Table 6 *Log-pricing errors of individual bond credit spreads.* This table shows how well the structural models match spreads of individual bonds. For a given rating r and maturity m, we find all actual bond spread observations in the sample period that have this rating and maturity, $s^a_{rm1}, ..., s^a_{rmN}$. For a given model, the corresponding model spreads (in basis points) are $s^M_{rm1}, ..., s^M_{rmN}$. The table shows $1/N \sum_{i=1}^{N} \Big| \log(s^a_{rmi}+1) - \log(s^M_{rmi}+1) \Big|$. 'Actual spread' is the actual spread (in basis points) to the swap rate. 'Short' includes bond maturities in the range 0-3 years, 'Medium' 3-10 years, and 'Long' 10-20 years. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. The sample period is 1988:03-2018:03.

		All	Short	Medium	Long
Inv	Actual spread	84	77	72	121
	BC-0G	91	63	97	110
		(50;141)	(26;109)	(50;160)	(81;132)
	BC	95	39***	91	173**
		(64;120)	(20;57)	(56;120)	(137;196)
	CDG	99	63	100	138
		(46;158)	(23;114)	(43;170)	(83;183)
	CDG-FL	75	61	84	70**
		(30;129)	(20;112)	(32;150)	(37;100)
	SD	112	61	112	171*
		(72;143)	(34;85)	(71;146)	(122;204)
	SD-FL	95	67	99	119
		(57;127)	(35;96)	(59;134)	(79;146)
Spec	Actual spread	413	409	387	470
	BC-0G	385	495	370	287***
		(225;539)	(229;776)	(226;498)	(217;345)
	BC	334	280*	349	366***
		(225;418)	(143;407)	(233;433)	(306;402)
	CDG	358	466	336	274***
		(178;537)	(189;772)	(175;487)	(170;361)
	CDG-FL	391	507	374	288***
		(192;577)	(204;827)	(193;532)	(177;375)
	SD	309**	331	314*	270***
		(206;385)	(198;438)	(214;387)	(201;317)
	SD-FL	385	433	389	320***
		(253;483)	(257;575)	(260;482)	(235;375)

Table 7 *Actual and model credit spreads.* This table shows average actual and model-implied corporate bond yield spreads (in basis points). Spreads are grouped according to remaining bond maturity at the spread observation date. 'Actual spread' is the actual spread to the swap rate. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. Confidence bands are simulation-based following Feldhütter and Schaefer (2018) and are at the 95% level. * implies significance at the 5% level and ** at the 1% level. The sample period is 1988-2018 for spread of bonds with a maturity more than three years and 2002-2018 for bonds with a maturity less than three years.

	$\hat{d}$	AAA	AA	A	BBB	BB	B	C
Actual spread		23	29	61	141	292	459	864
BC-0G								
Benchmark	0.86	11**	13**	79	145	270	505	557**
		(5;18)	(7;18)	(49;104)	(94;185)	(176;341)	(337;627)	(382;697)
BGY adjustment	0.71	4**	6**	44**	81**	166**	355*	572**
		(2;6)	(4;9)	(28;57)	(54;102)	(108;210)	(237;441)	(404;705)
BFKS adjustment	0.87	9**	12**	74	130	266	547	1005
		(4;15)	(7;17)	(46;97)	(84;167)	(173;336)	(368;675)	(592;1123)
CDG								
Benchmark	0.89	15*	22	103*	180	299	503	565**
		(8;22)	(14;29)	(71;128)	(128;218)	(211;361)	(361;599)	(427;657)
BGY adjustment	0.77	8**	16**	77	134	239*	442	657**
		(4;12)	(10;21)	(53;96)	(95;162)	(167;289)	(317;527)	(508;753)
BFKS adjustment	0.90	14*	24	106**	178	317	573	803
		(7;21)	(15;31)	(72;131)	(125;216)	(223;383)	(416;677)	(627;923)
SD-FL								
Benchmark	0.63	16	23	92	150	255	451	464**
		(6;28)	(10;36)	(48;131)	(83;210)	(140;350)	(256;607)	(271;634)
BGY adjustment	0.57	6**	12**	53	87**	159**	318*	476**
		(2;11)	(5;19)	(28;76)	(48;121)	(86;221)	(182;431)	(277;684)
BFKS adjustment	0.66	14	23	88	139	252	487	816
		(5;24)	(10;36)	(46;125)	(76;193)	(139;345)	(281;652)	(420;1010)

Table 8 *Model spreads using different approaches.* This table shows the average actual and model-implied spreads for bonds with a maturity between 3–20 years. 'Actual spread' is the actual spread to the swap rate. 'BC-0G' is the Black-Cox model with zero growth in debt. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD-FL' refers to a model where the firm adjusts the level of debt such that a long-run leverage, firm specific and calculated as the historical average firm leverage, is targeted, and the level of debt is subject to random shocks. 'Benchmark' refers to the results in the main section when assuming that the market value of debt is equal to the book value of debt. 'BGY adjustment' shows the results when using the adjustment to leverage suggested in Bai, Goldstein, and Yang (2020), while 'BFKS adjustment' shows the results when using the adjustment to leverage suggested by Bretscher, Feldhütter, Kane, and Schmid (2020). Confidence bands are simulation-based following Feldhütter and Schaefer (2018) and are at the 95% level.* implies significance at the 5% level and ** at the 1% level. The sample period is 1988:03-2018:03.

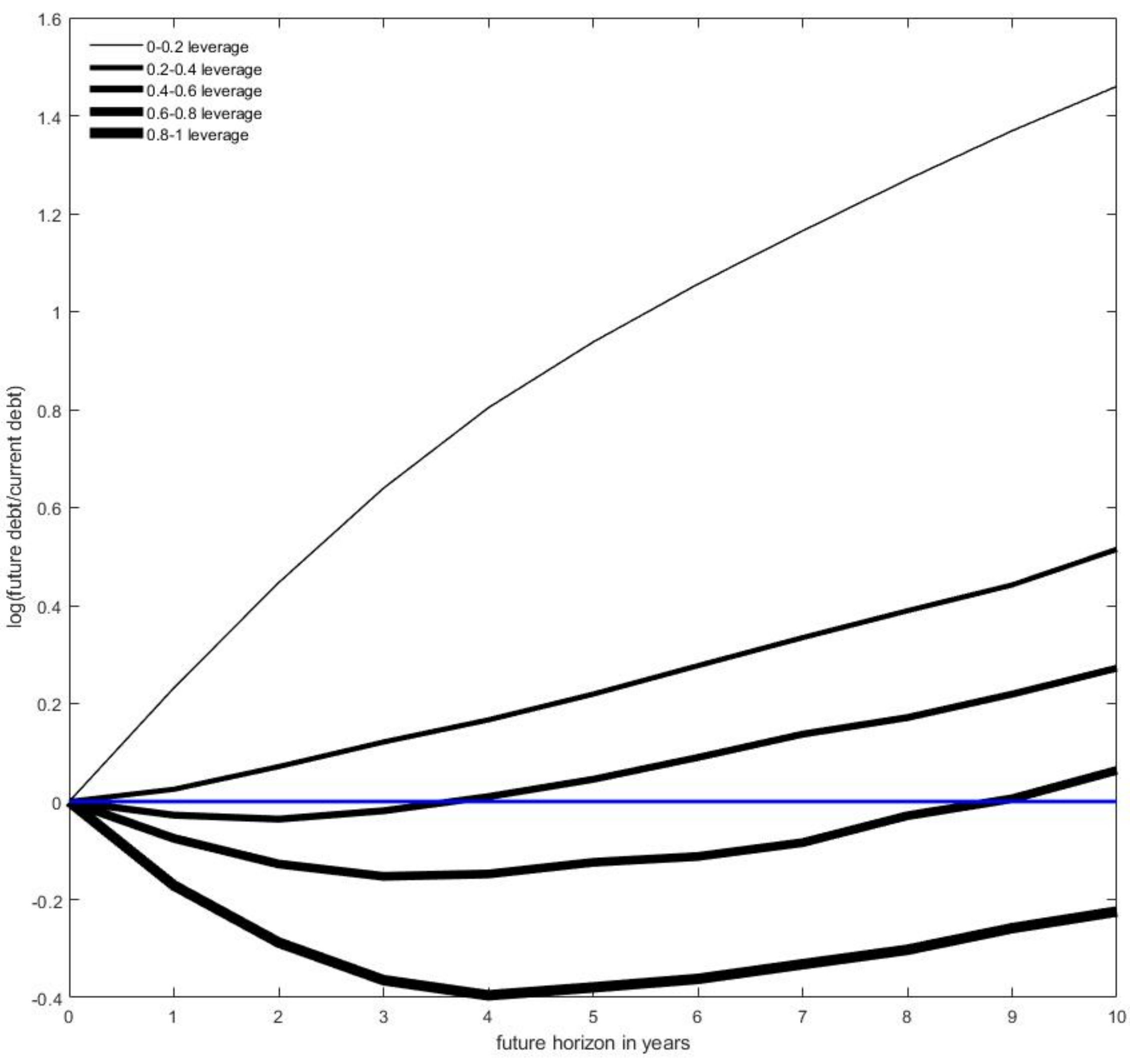

Fig. 2 *Future debt growth as a function of initial leverage.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. The figure shows the average ratio for different initial leverage ratios and future horizons. The data is from CRSP/Compustat and the sample period is 1965–2017.

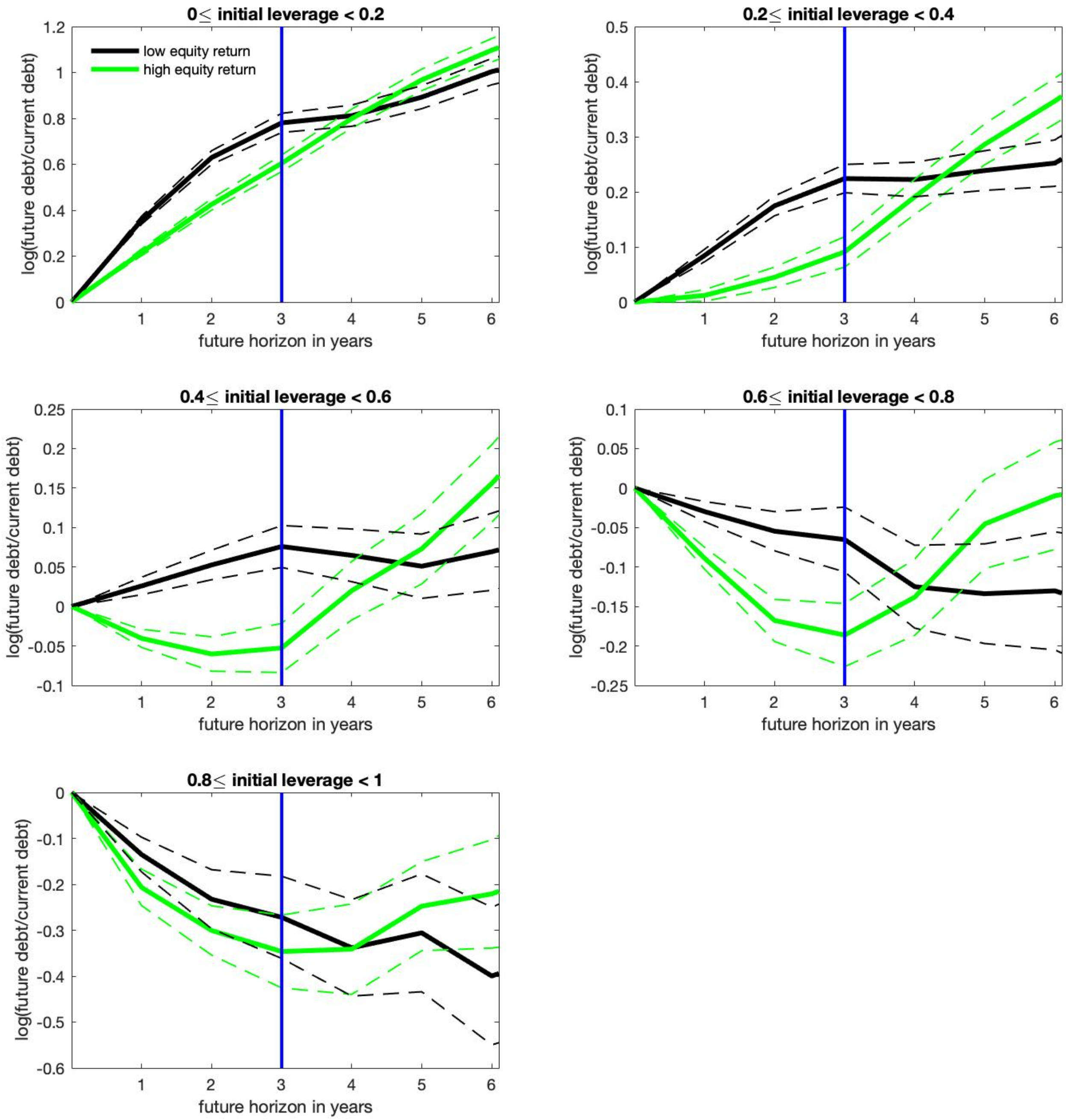

Fig. 3 *Future debt growth conditional on future three-year equity returns for different leverage ratios.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The figure shows the average log-ratio for high and low future equity return firms. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1965-2017.

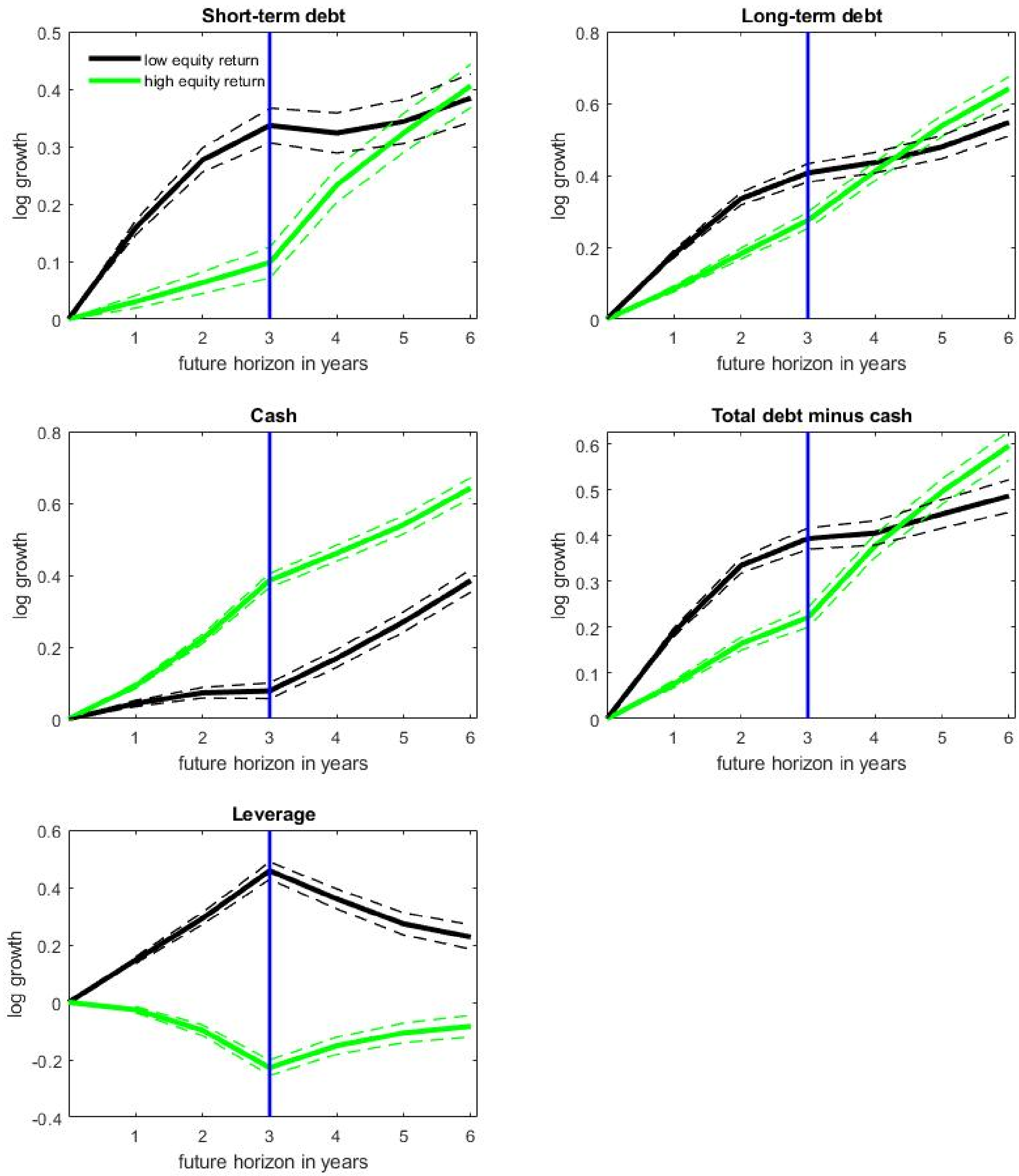

Fig. 4 *Future growth in short-term debt, long-term debt, cash, and leverage conditional on future three-year equity returns.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{B_{i,t+T}}{B_{i,t}}\right)$ where $B_{i,t}$ is the variable of interest for firm i in year t and T is the horizon in years. For each firm-year in the sample, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The figure shows the average log-ratio for high and low future equity return firms. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1965-2017.

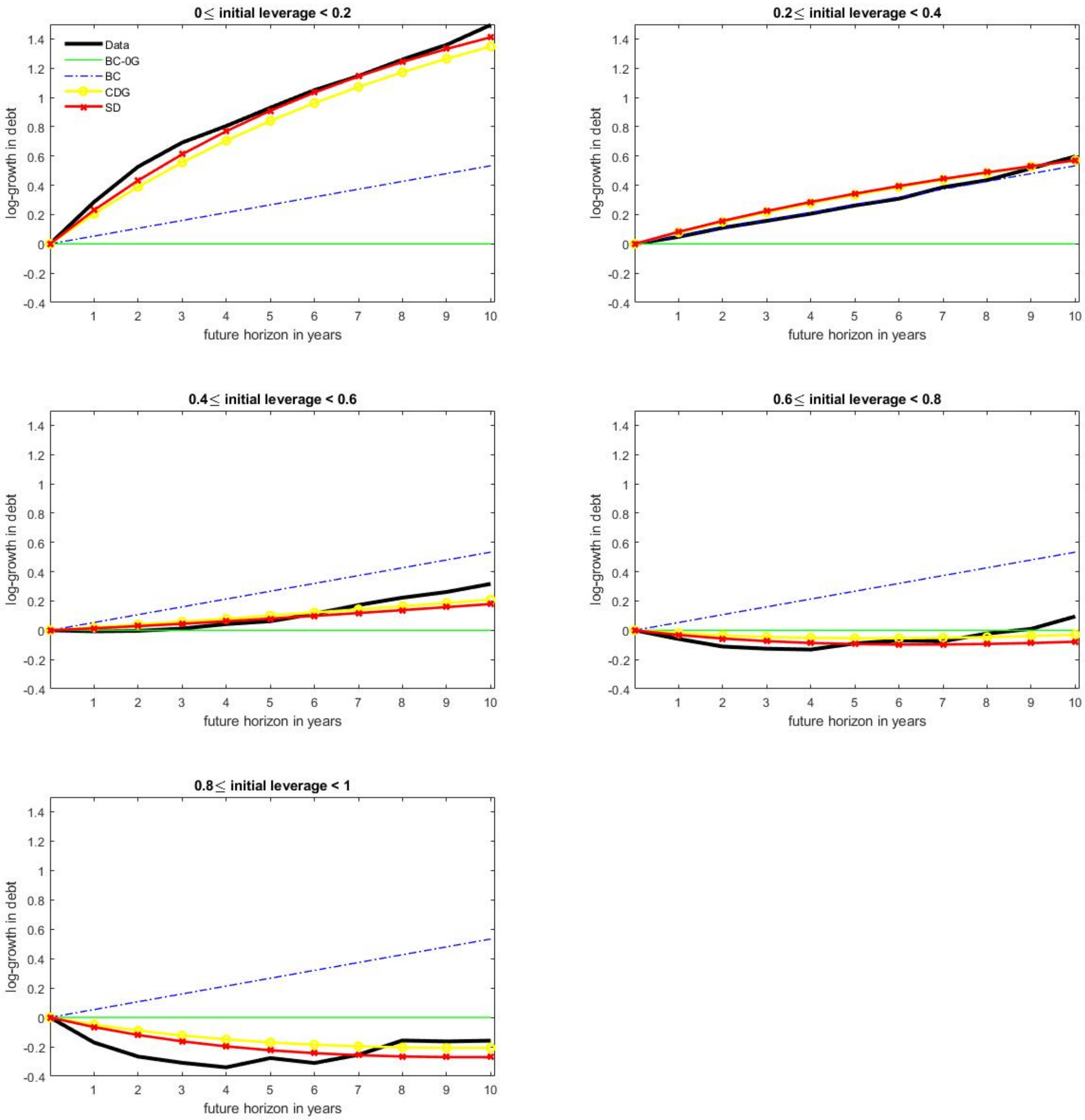

Fig. 5 *Future debt relative to current debt, model-fit.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. 'Data' shows the average log-ratio for different initial leverage ratios and future horizons in the data. The figure also shows fitted values from structural models. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. The data is from CRSP/Compustat and the sample period is 1965-2017.

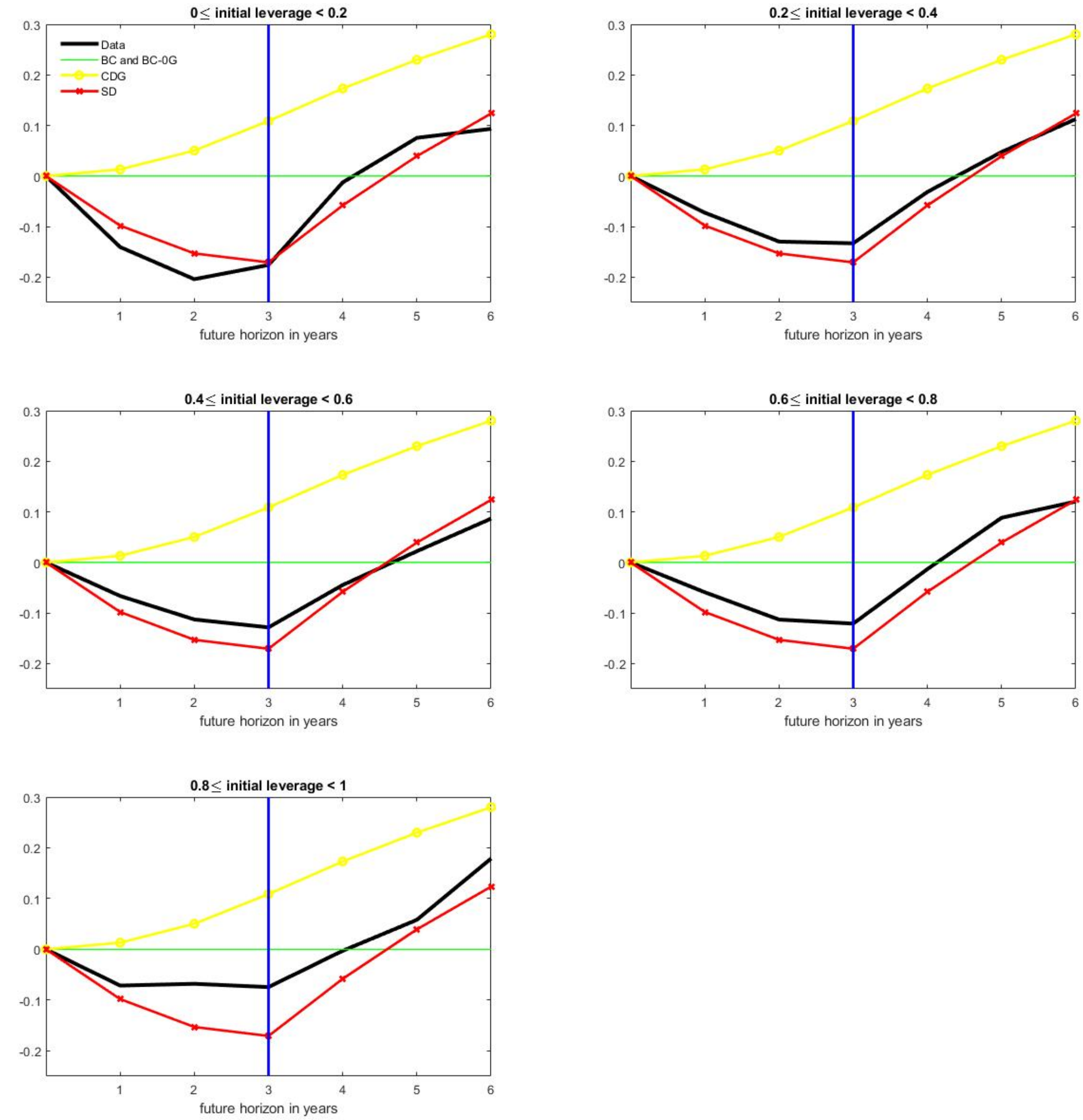

Fig. 6 *Future debt of firms with high future three-year equity returns minus future debt of firms with low three-year equity returns.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The figure shows for both the fitted structural models and the data the difference in average increase in log-debt for high and low future equity return firms. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. The data is from CRSP/Compustat and the sample period is 1965-2017.

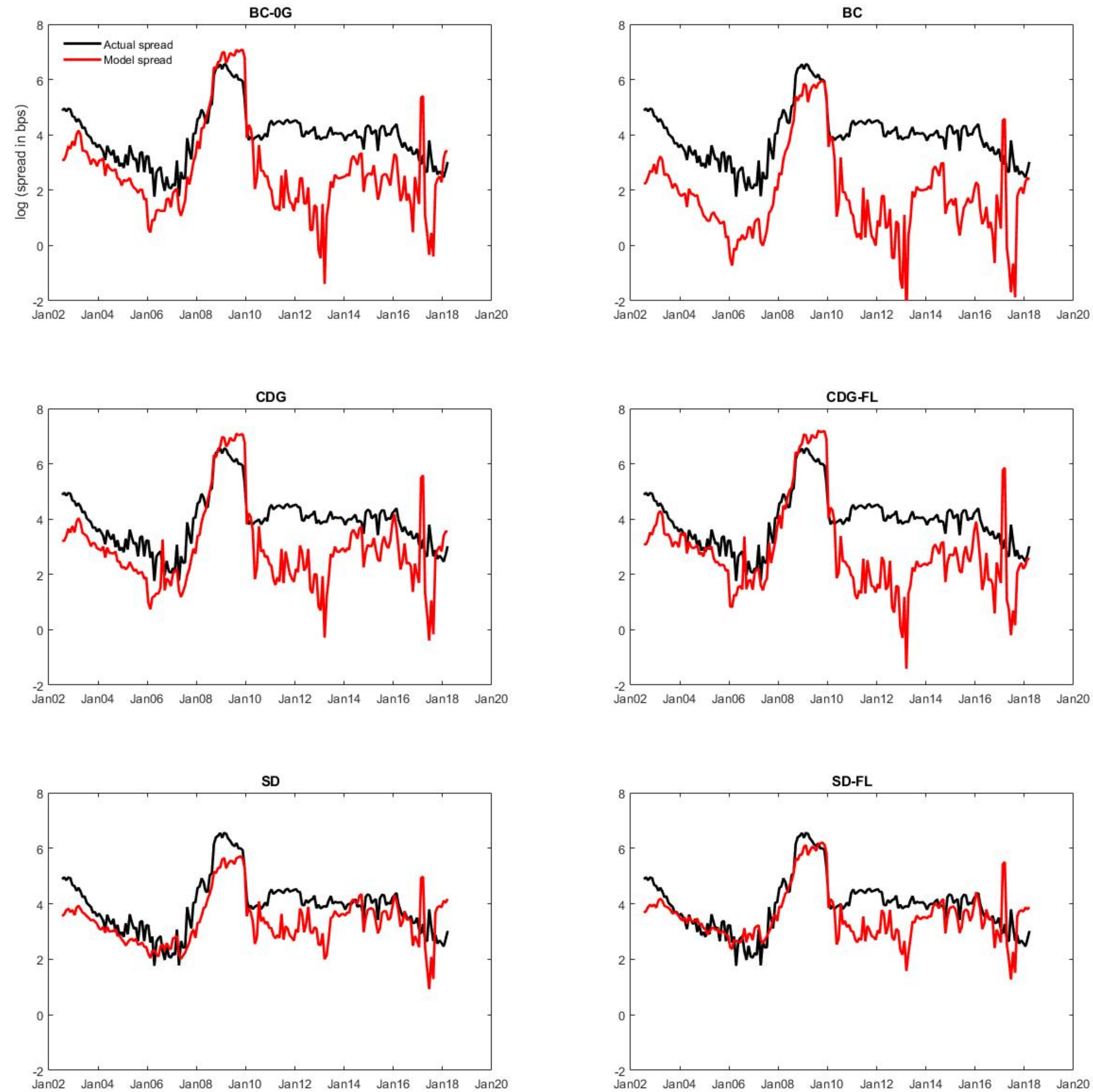

Fig. 7 *Actual and model short term investment grade credit spreads.* For investment grade rating (rated BBB- or higher) and bond maturity less than three years, we find all bonds at the end of a given month t that have this rating and maturity, calculate the average actual credit spread s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of monthly average model credit spread $s^M_{rm1}, ..., s^M_{rmT}$. This figure shows the time series of log credit spreads.

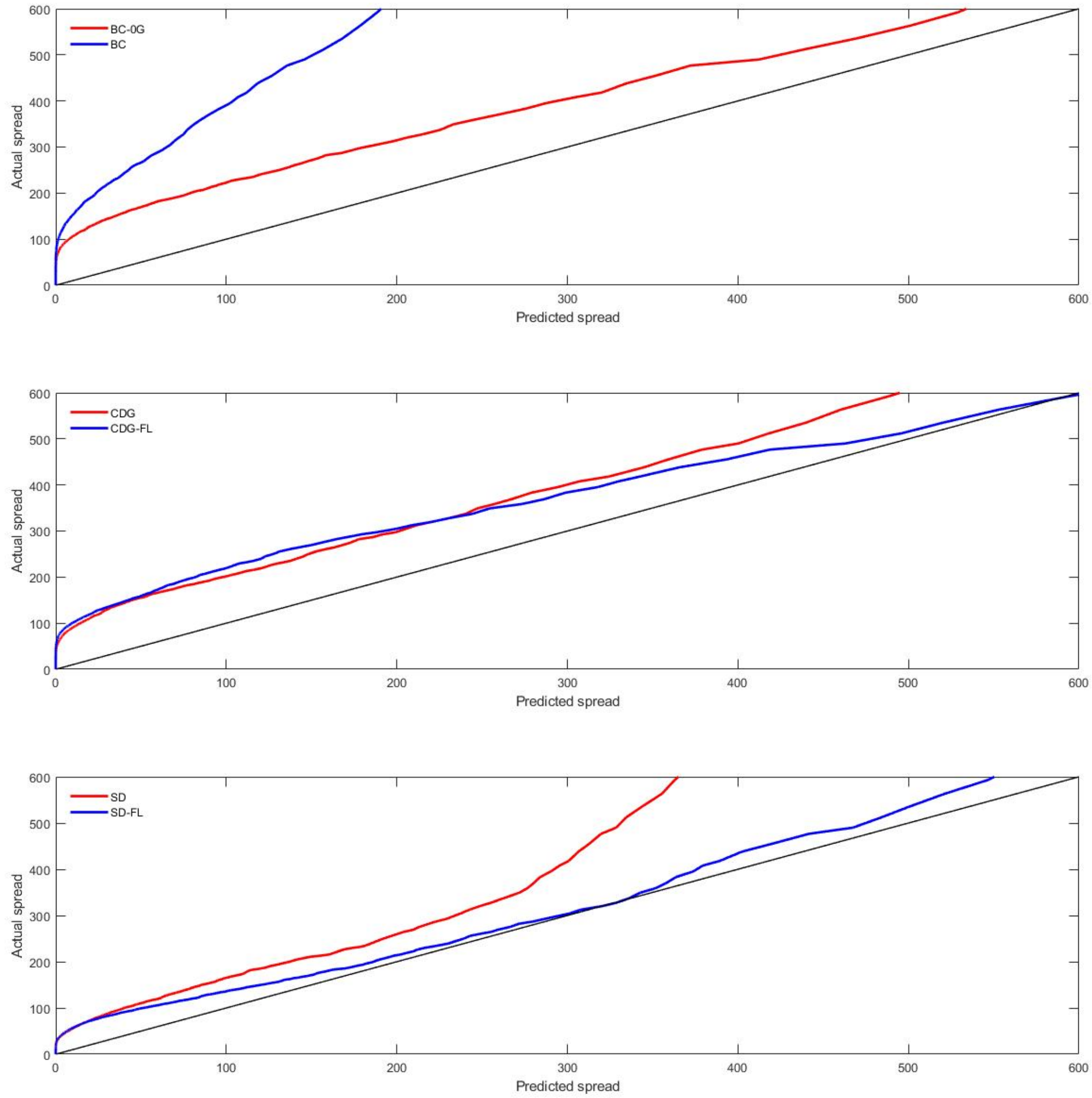

Fig. 8 *QQ-plot of predicted credit spreads against actual credit spreads for short-term investment grade bonds.* The graphs plot the quantiles of model-implied credit spreads against actual credit spreads for bonds with an investment grade rating and bond maturity less than three years. 'Actual spread' is the actual spread to the swap rate. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. The sample period is 2002:07-2018:03.

horizon (years)	1	2	3	4	5	6	7	8	9	10
Leverage 0-0.2										
Data	0.29	0.53	0.69	0.81	0.93	1.05	1.15	1.26	1.36	1.50
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.05	0.11	0.16	0.21	0.27	0.32	0.37	0.43	0.48	0.53
CDG	0.21	0.39	0.56	0.71	0.84	0.96	1.07	1.17	1.26	1.35
SD	0.23	0.43	0.61	0.77	0.91	1.03	1.14	1.24	1.33	1.41
Leverage 0.2-0.4										
Data	0.05	0.11	0.16	0.21	0.26	0.31	0.39	0.44	0.52	0.60
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.05	0.11	0.16	0.21	0.27	0.32	0.37	0.43	0.48	0.53
CDG	0.08	0.15	0.22	0.28	0.33	0.39	0.44	0.48	0.53	0.57
SD	0.08	0.16	0.22	0.29	0.34	0.40	0.44	0.49	0.53	0.57
Leverage 0.4-0.6										
Data	-0.01	-0.00	0.01	0.04	0.06	0.11	0.17	0.22	0.26	0.32
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.05	0.11	0.16	0.21	0.27	0.32	0.37	0.43	0.48	0.53
CDG	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.19	0.21
SD	0.01	0.03	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
Leverage 0.6-0.8										
Data	-0.06	-0.11	-0.13	-0.13	-0.09	-0.07	-0.07	-0.02	0.01	0.09
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.05	0.11	0.16	0.21	0.27	0.32	0.37	0.43	0.48	0.53
CDG	-0.02	-0.03	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.04	-0.03
SD	-0.03	-0.06	-0.07	-0.09	-0.09	-0.10	-0.10	-0.09	-0.09	-0.08
Leverage 0.8-1										
Data	-0.17	-0.27	-0.31	-0.34	-0.28	-0.31	-0.25	-0.16	-0.16	-0.16
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.05	0.11	0.16	0.21	0.27	0.32	0.37	0.43	0.48	0.53
CDG	-0.05	-0.09	-0.12	-0.15	-0.17	-0.19	-0.20	-0.20	-0.21	-0.21
SD	-0.07	-0.12	-0.16	-0.20	-0.22	-0.24	-0.26	-0.27	-0.27	-0.27

Table A1 *Log ratio of future debt relative to current debt, model-fit.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. 'Data' shows the average log-ratio for different initial leverage ratios and future horizons in the data. The table also shows fitted values from structural models. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks.

horizon (years)	1	2	3	4	5	6	7	8	9	10
Leverage 0-0.2										
Data	-0.14	-0.20	-0.18	-0.01	0.08	0.09	0.13	0.17	0.16	0.14
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.01	0.05	0.11	0.17	0.23	0.28	0.32	0.36	0.40	0.43
SD	-0.10	-0.15	-0.17	-0.06	0.04	0.12	0.20	0.26	0.31	0.36
Leverage 0.2-0.4										
Data	-0.07	-0.13	-0.13	-0.03	0.05	0.11	0.12	0.11	0.14	0.13
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.01	0.05	0.11	0.17	0.23	0.28	0.32	0.36	0.40	0.43
SD	-0.10	-0.15	-0.17	-0.06	0.04	0.12	0.20	0.26	0.31	0.36
Leverage 0.4-0.6										
Data	-0.07	-0.11	-0.13	-0.04	0.02	0.09	0.16	0.17	0.17	0.20
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.01	0.05	0.11	0.17	0.23	0.28	0.32	0.36	0.40	0.43
SD	-0.10	-0.15	-0.17	-0.06	0.04	0.12	0.20	0.26	0.31	0.36
Leverage 0.6-0.8										
Data	-0.06	-0.11	-0.12	-0.01	0.09	0.12	0.17	0.15	0.11	0.12
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.01	0.05	0.11	0.17	0.23	0.28	0.32	0.36	0.40	0.43
SD	-0.10	-0.15	-0.17	-0.06	0.04	0.12	0.20	0.26	0.31	0.36
Leverage 0.8-1										
Data	-0.07	-0.07	-0.07	-0.00	0.06	0.18	0.18	0.21	0.16	0.15
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.01	0.05	0.11	0.17	0.23	0.28	0.32	0.36	0.40	0.43
SD	-0.10	-0.15	-0.17	-0.06	0.04	0.12	0.20	0.26	0.31	0.36

Table A2 *Log ratio of future debt relative to current debt conditional on future equity returns, model-fit.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The table shows the difference in log-ratio for high and low future equity return firms. The table also shows fitted values from structural models. 'BC-0G' to the Black-Cox model with zero growth in debt. 'BC' refers to the Black-Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks.

horizon (years)	horizon (years) 1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	obs.
AAA																
Moody's report	0.00	0.01	0.01	0.03	0.08	0.14	0.19	0.25	0.31	0.38	0.52	0.64	0.73	0.80	0.80	
All firms, cohort-weight	0.00	0.01	0.01	0.02	0.06	0.10	0.14	0.18	0.22	0.27	0.37	0.46	0.52	0.57	0.57	85708
US industrial firms, cohort-weight	0.00	0.00	0.00	0.07	0.24	0.41	0.59	0.77	0.95	1.14	1.55	1.86	2.10	2.26	2.26	17019
US industrial firms, equal-weight	**0.00**	**0.00**	**0.00**	**0.06**	**0.16**	**0.29**	**0.42**	**0.51**	**0.62**	**0.72**	**0.96**	**1.17**	**1.34**	**1.46**	**1.46**	**17019**
AA																
Moody's report	0.02	0.06	0.11	0.19	0.29	0.40	0.52	0.63	0.71	0.79	1.02	1.29	1.49	1.78	2.26	
All firms, cohort-weight	0.02	0.08	0.15	0.25	0.38	0.49	0.62	0.74	0.86	0.98	1.29	1.61	1.86	2.19	2.69	262844
US industrial firms, cohort-weight	0.00	0.01	0.04	0.18	0.31	0.40	0.47	0.55	0.62	0.66	0.83	1.18	1.42	1.71	2.35	43725
US industrial firms, equal-weight	**0.00**	**0.01**	**0.02**	**0.12**	**0.21**	**0.29**	**0.35**	**0.42**	**0.48**	**0.51**	**0.64**	**0.96**	**1.19**	**1.49**	**2.16**	**43725**
A																
Moody's report	0.05	0.16	0.33	0.52	0.74	0.99	1.26	1.54	1.85	2.15	2.74	3.36	4.08	4.82	5.53	
All firms, cohort-weight	0.06	0.17	0.35	0.55	0.79	1.06	1.35	1.65	1.95	2.26	2.90	3.57	4.35	5.14	5.87	517501
US industrial firms, cohort-weight	0.03	0.12	0.27	0.41	0.60	0.84	1.08	1.33	1.58	1.84	2.43	3.07	3.76	4.56	5.33	135673
US industrial firms, equal-weight	**0.02**	**0.10**	**0.22**	**0.35**	**0.52**	**0.74**	**0.96**	**1.20**	**1.44**	**1.70**	**2.28**	**2.91**	**3.60**	**4.42**	**5.22**	**135673**
BBB																
Moody's report	0.17	0.44	0.77	1.16	1.55	1.95	2.35	2.77	3.24	3.75	4.88	6.07	7.29	8.56	9.65	
All firms, cohort-weight	0.18	0.48	0.82	1.22	1.62	2.02	2.42	2.85	3.32	3.83	4.93	6.13	7.40	8.64	9.76	494030
US industrial firms, cohort-weight	0.18	0.52	0.97	1.54	2.11	2.73	3.36	4.09	4.91	5.74	7.42	9.17	11.07	13.05	14.82	163580
US industrial firms, equal-weight	**0.16**	**0.50**	**0.98**	**1.59**	**2.21**	**2.91**	**3.66**	**4.53**	**5.49**	**6.43**	**8.26**	**10.09**	**11.95**	**13.99**	**15.83**	**163580**
BB																
Moody's report	0.92	2.52	4.38	6.36	8.20	9.90	11.39	12.85	14.34	15.88	18.81	21.54	24.31	26.65	28.71	
All firms, cohort-weight	0.92	2.43	4.17	6.05	7.79	9.39	10.85	12.26	13.70	15.16	17.91	20.44	23.06	25.19	27.07	282165
US industrial firms, cohort-weight	1.11	2.99	5.12	7.37	9.52	11.63	13.59	15.50	17.44	19.42	23.49	27.56	31.93	35.60	38.50	135738
US industrial firms, equal-weight	**1.05**	**2.80**	**4.83**	**7.07**	**9.37**	**11.79**	**14.12**	**16.39**	**18.64**	**20.85**	**25.79**	**30.70**	**35.79**	**39.90**	**42.80**	**135738**
B																
Moody's report	3.45	8.15	12.96	17.32	21.31	24.92	28.17	30.92	33.42	35.51	38.76	41.71	44.52	46.87	48.71	
All firms, cohort-weight	3.50	8.13	12.83	17.06	20.94	24.42	27.57	30.23	32.59	34.64	37.78	40.64	43.45	45.87	47.69	333112
US industrial firms, cohort-weight	3.81	8.87	14.04	18.65	22.96	26.85	30.38	33.46	36.22	38.70	42.79	46.76	50.61	53.89	56.93	203217
US industrial firms, equal-weight	**4.43**	**9.15**	**13.86**	**18.08**	**22.33**	**26.28**	**30.00**	**33.40**	**36.58**	**40.53**	**45.68**	**50.45**	**55.06**	**57.16**	**58.81**	**203217**
C																
Moody's report	10.22	18.04	24.64	30.17	34.67	38.09	41.12	44.04	46.76	48.90	51.35	51.84	52.43	52.53	52.53	
All firms, cohort-weight	9.09	16.03	22.04	26.97	31.01	34.16	36.83	39.24	41.33	42.82	45.11	46.11	47.34	47.97	49.34	203057
US industrial firms, cohort-weight	8.96	16.19	22.60	27.97	32.55	36.04	39.08	42.44	46.23	49.34	54.24	56.39	59.27	61.68	64.32	133834
US industrial firms, equal-weight	**8.39**	**14.78**	**20.70**	**26.06**	**31.27**	**35.07**	**38.09**	**41.36**	**44.96**	**48.18**	**56.51**	**61.50**	**65.04**	**66.36**	**67.76**	**133834**

Table A3 *Average cumulative default rates, 1970–2017.* 'Moody's report' refers to default rates for 1970–2017 published in Moody's (2018). 'All firms, cohort-weight' is the calculated default rate when using Moody's methodology and their default database. 'US industrial firms, cohort-weight' is the calculated default rate when using Moody's methodology and restricting the sample to US industrial firms in their default database. 'US industrial firms, equal-weight' is the calculated default rate when using Moody's methodology and restricting the sample to US industrial firms in their default database, with one change applied to their methodology: default rates from different cohorts are weighted equally instead of weighted by the cohort size. Cohorts are formed on a monthly basis and 'observations' is the sum of all cohort sizes.

	horizon (years)													obs.
	1	2	3	4	5	6	8	10	12	14	16	18	20	
AAA														
US industrial firms	0.00	0.00	0.00	0.06	0.16	0.29	0.51	0.72	0.96	1.17	1.34	1.46	1.46	16344
Firms that are not US industrials	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	65892
Difference	0.00	-0.00	-0.00	0.05	0.15	0.29	0.51*	0.72*	0.95*	1.16	1.33	1.46	1.46	
AA														
US industrial firms	0.00	0.01	0.02	0.12	0.21	0.29	0.42	0.51	0.64	0.96	1.19	1.49	2.16	42162
Firms that are not US industrials	0.02	0.06	0.12	0.20	0.32	0.47	0.84	1.08	1.37	1.65	1.82	2.10	2.37	207636
Difference	-0.02	-0.05	-0.10	-0.08	-0.11	-0.18	-0.42	-0.57	-0.72	-0.69	-0.64	-0.60	-0.21	
A														
US industrial firms	0.02	0.10	0.22	0.35	0.52	0.74	1.20	1.70	2.28	2.91	3.60	4.42	5.22	128700
Firms that are not US industrials	0.04	0.14	0.31	0.50	0.69	0.90	1.38	1.92	2.43	2.92	3.61	4.38	5.13	355512
Difference	-0.02	-0.04	-0.09	-0.15	-0.17	-0.16	-0.18	-0.22	-0.15	-0.02	-0.01	0.05	0.10	
BBB														
US industrial firms	0.16	0.50	0.98	1.59	2.21	2.91	4.53	6.43	8.26	10.09	11.95	13.99	15.83	151602
Firms that are not US industrials	0.16	0.41	0.71	1.06	1.41	1.69	2.14	2.61	3.30	4.21	5.09	5.87	6.55	306102
Difference	0.00	0.08	0.27	0.52	0.80	1.22*	2.38**	3.82**	4.96**	5.88***	6.85***	8.12***	9.27***	
BB														
US industrial firms	1.05	2.80	4.83	7.07	9.37	11.79	16.39	20.85	25.79	30.70	35.79	39.90	42.80	121470
Firms that are not US industrials	0.93	2.26	3.81	5.44	6.88	8.01	9.98	11.87	13.45	14.60	15.74	16.80	18.58	131898
Difference	0.11	0.54	1.02	1.63	2.49	3.77*	6.41**	8.98***	12.33***	16.10***	20.05***	23.10***	24.22***	
B														
US industrial firms	4.43	9.15	13.86	18.08	22.33	26.28	33.40	40.53	45.68	50.45	55.06	57.16	58.81	174288
Firms that are not US industrials	4.22	8.75	12.91	16.46	19.63	22.57	26.88	30.34	32.08	34.69	37.51	39.62	40.68	109848
Difference	0.21	0.40	0.95	1.62	2.70	3.70	6.52**	10.19***	13.59***	15.76***	17.55***	17.54***	18.12**	
C														
US industrial firms	8.39	14.78	20.70	26.06	31.27	35.07	41.36	48.18	56.51	61.50	65.04	66.36	67.76	98364
Firms that are not US industrials	7.72	12.46	16.77	20.61	23.71	26.95	30.95	31.81	32.47	32.80	33.10	33.11	33.24	50310
Difference	0.67	2.33	3.92	5.45	7.56	8.13	10.41	16.36	24.05**	28.70**	31.94**	33.25*	34.52**	

Table A4 *Average cumulative default rates of US industrials vs the rest, 1970–2017.* 'US industrial firms' is the calculated default rate when using Moody's methodology and restricting the sample to US industrial firms in their default database, with one change applied to their methodology: default rates from different cohorts are weighted equally instead of weighted by the cohort size. 'Firms that are not US industrials' refers to the calculated default rate using all observations in the Moody's default database that are not industrial firms from the US. Cohorts are formed on a monthly basis and 'observations' is the sum of all cohort sizes. '*', '**', and '***' show when the difference is outside the 95%, 99%, and 99.9% confidence band, respectively, where the confidence bands are calculated according to Appendix B.3.

	#firms	Mean	10th	25th	Median	75th	90th
			Leverage ratio				
AAA	16	0.12	0.03	0.05	0.07	0.14	0.25
AA	68	0.17	0.07	0.11	0.17	0.20	0.24
A	232	0.28	0.11	0.17	0.25	0.36	0.50
BBB	355	0.35	0.15	0.23	0.32	0.44	0.55
BB	208	0.44	0.20	0.31	0.42	0.56	0.68
B	123	0.56	0.25	0.38	0.58	0.71	0.87
C	51	0.69	0.40	0.56	0.72	0.87	0.94
all	575	0.32	0.11	0.18	0.29	0.44	0.60
			Equity volatility				
AAA	16	0.25	0.16	0.19	0.23	0.29	0.36
AA	68	0.25	0.16	0.20	0.24	0.29	0.38
A	232	0.31	0.19	0.24	0.29	0.36	0.41
BBB	355	0.33	0.21	0.25	0.31	0.39	0.47
BB	208	0.44	0.27	0.31	0.40	0.50	0.62
B	123	0.55	0.28	0.35	0.48	0.68	0.85
C	51	0.61	0.36	0.44	0.56	0.74	0.98
all	575	0.34	0.20	0.24	0.31	0.40	0.50
			Asset volatility				
AAA	16	0.22	0.18	0.20	0.20	0.24	0.25
AA	68	0.22	0.20	0.20	0.22	0.24	0.25
A	232	0.24	0.18	0.20	0.23	0.25	0.29
BBB	355	0.24	0.16	0.20	0.23	0.27	0.34
BB	208	0.26	0.17	0.21	0.26	0.31	0.34
B	123	0.27	0.17	0.22	0.25	0.30	0.37
C	51	0.26	0.11	0.19	0.24	0.31	0.40
all	575	0.24	0.17	0.20	0.23	0.26	0.32
			Payout ratio				
AAA	16	0.042	0.012	0.020	0.044	0.059	0.072
AA	68	0.037	0.011	0.018	0.034	0.050	0.065
A	232	0.041	0.017	0.023	0.037	0.052	0.070
BBB	355	0.047	0.017	0.028	0.042	0.059	0.085
BB	208	0.046	0.018	0.027	0.040	0.055	0.077
B	123	0.049	0.022	0.033	0.044	0.060	0.079
C	51	0.051	0.026	0.039	0.049	0.059	0.072
all	575	0.044	0.016	0.025	0.040	0.055	0.076

Table A5 *Firm summary statistics.* For each bond yield observation, the leverage ratio, equity volatility, asset volatility, and payout ratio are calculated for the issuing firm on the day of the observation. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility, calculated as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Firm variables are computed using data from CRSP and Compustat.